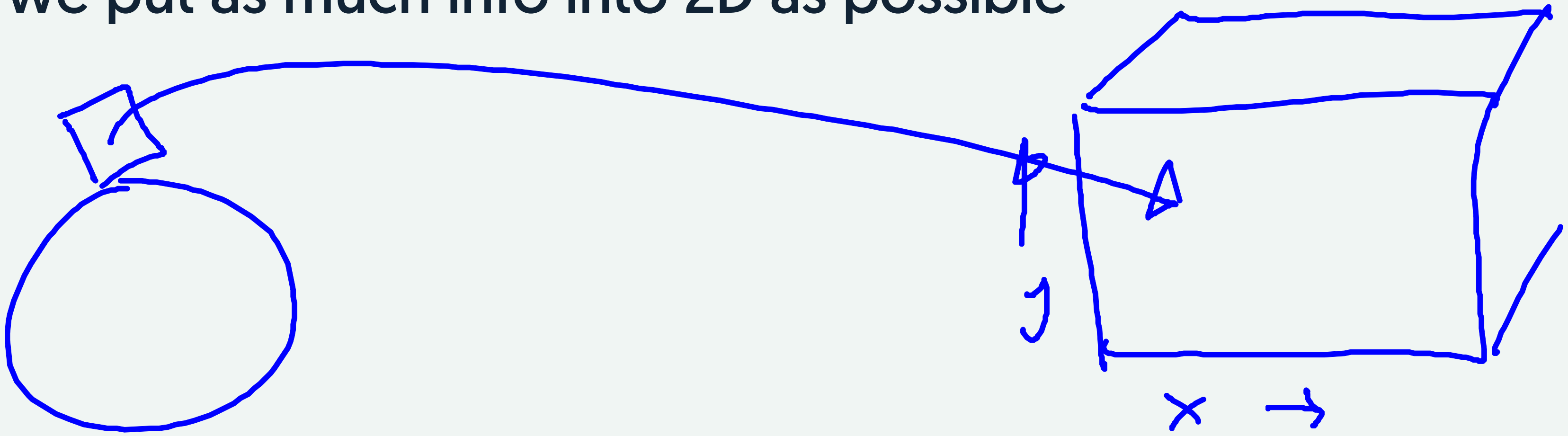
Projections
Perspective Math
And Programming

Projection 3D to 2D

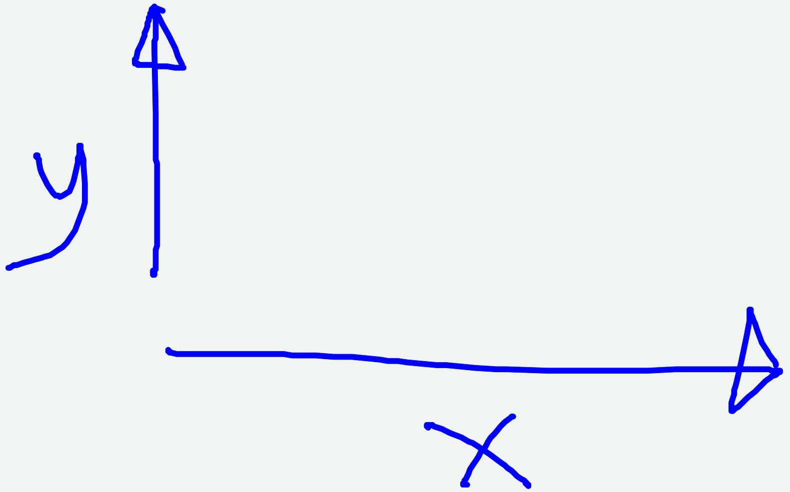
We lose a dimension

- No - we actually keep it (screen as a fishtank)
- Yes - we put as much info into 2D as possible



The Screen as a Fishtank

- X and Y (front/back)
- Z into screen
  - negative Z into screen

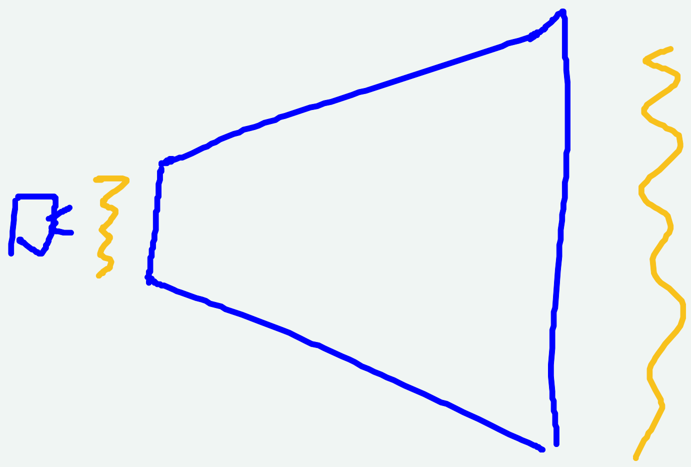


Near and Far

Some things are too close (in front of fishtank)

Some things are too far (behind fishtank)

Need to limit things for technical reasons

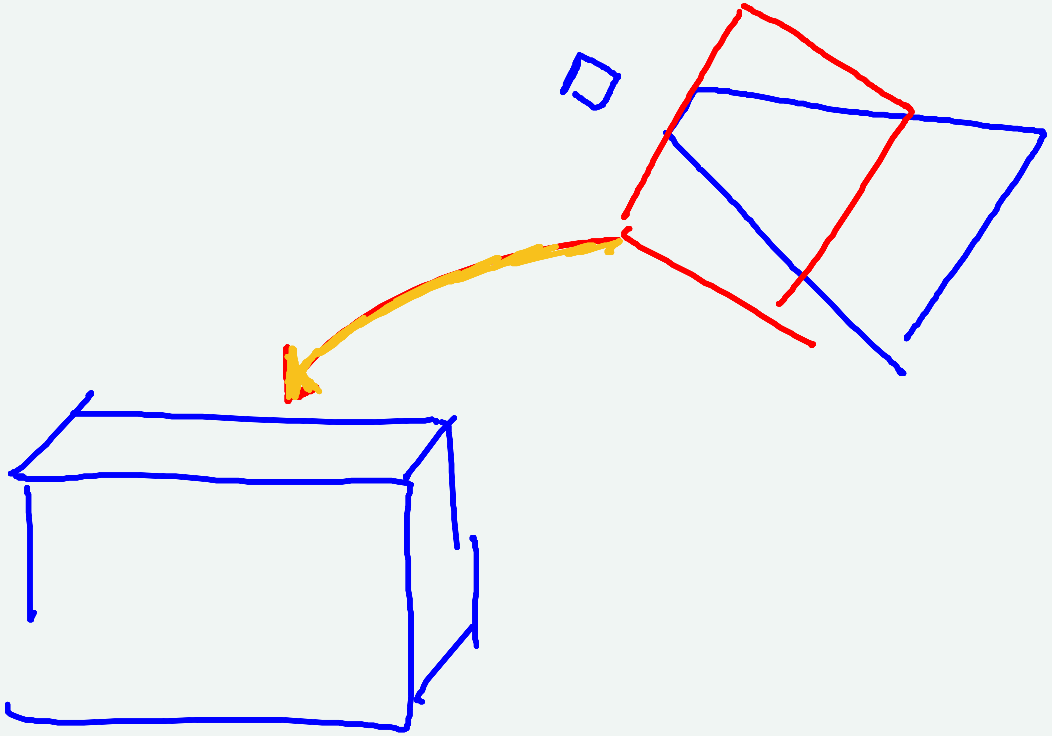


Projection

From: What is in front of camera

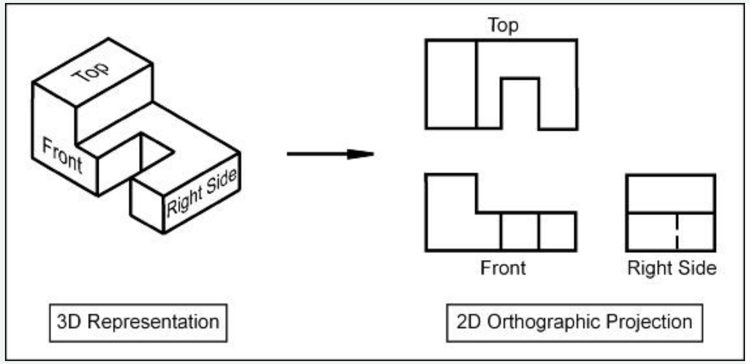
To: Screen Fishtank

It's a **transformation**

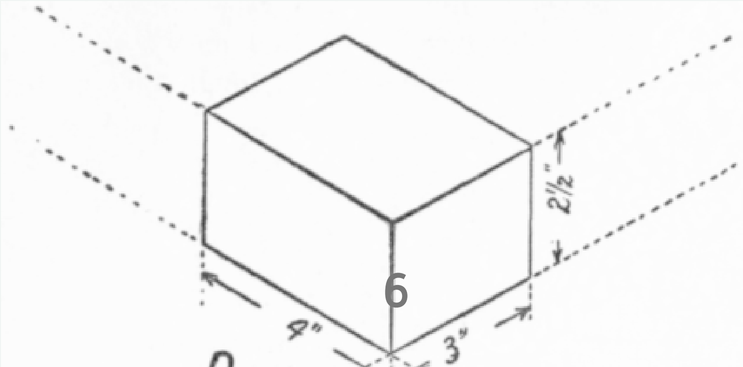


Types of Projections

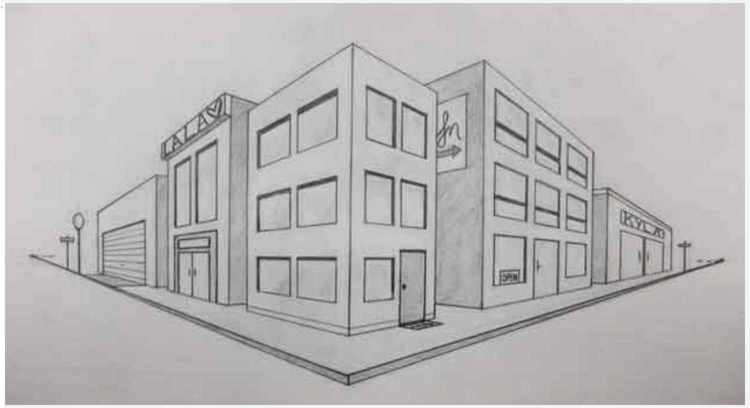
Orthographic



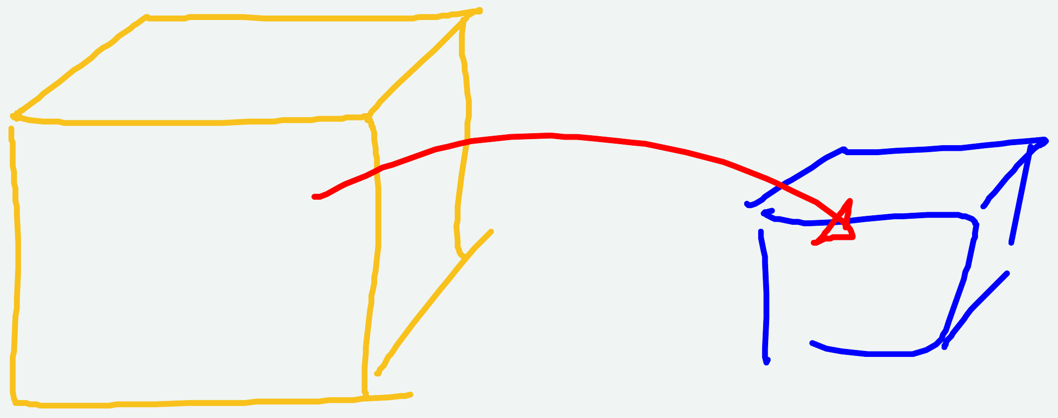
Isometric



Perspective



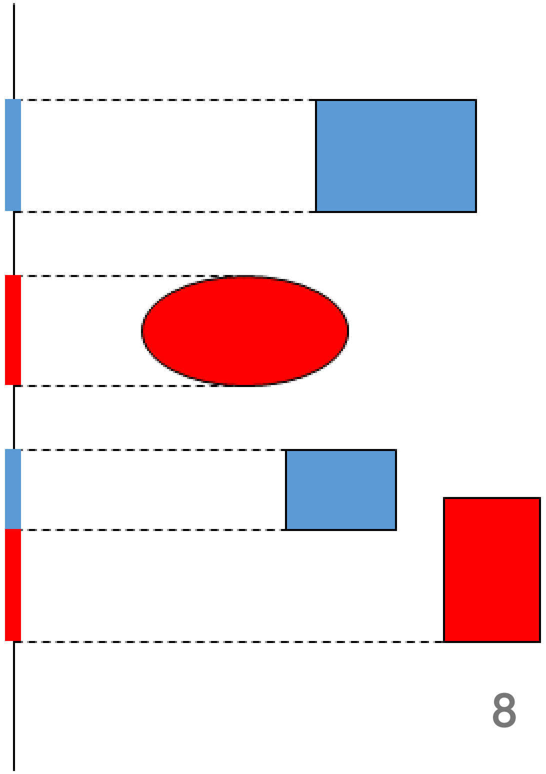
Orthographic



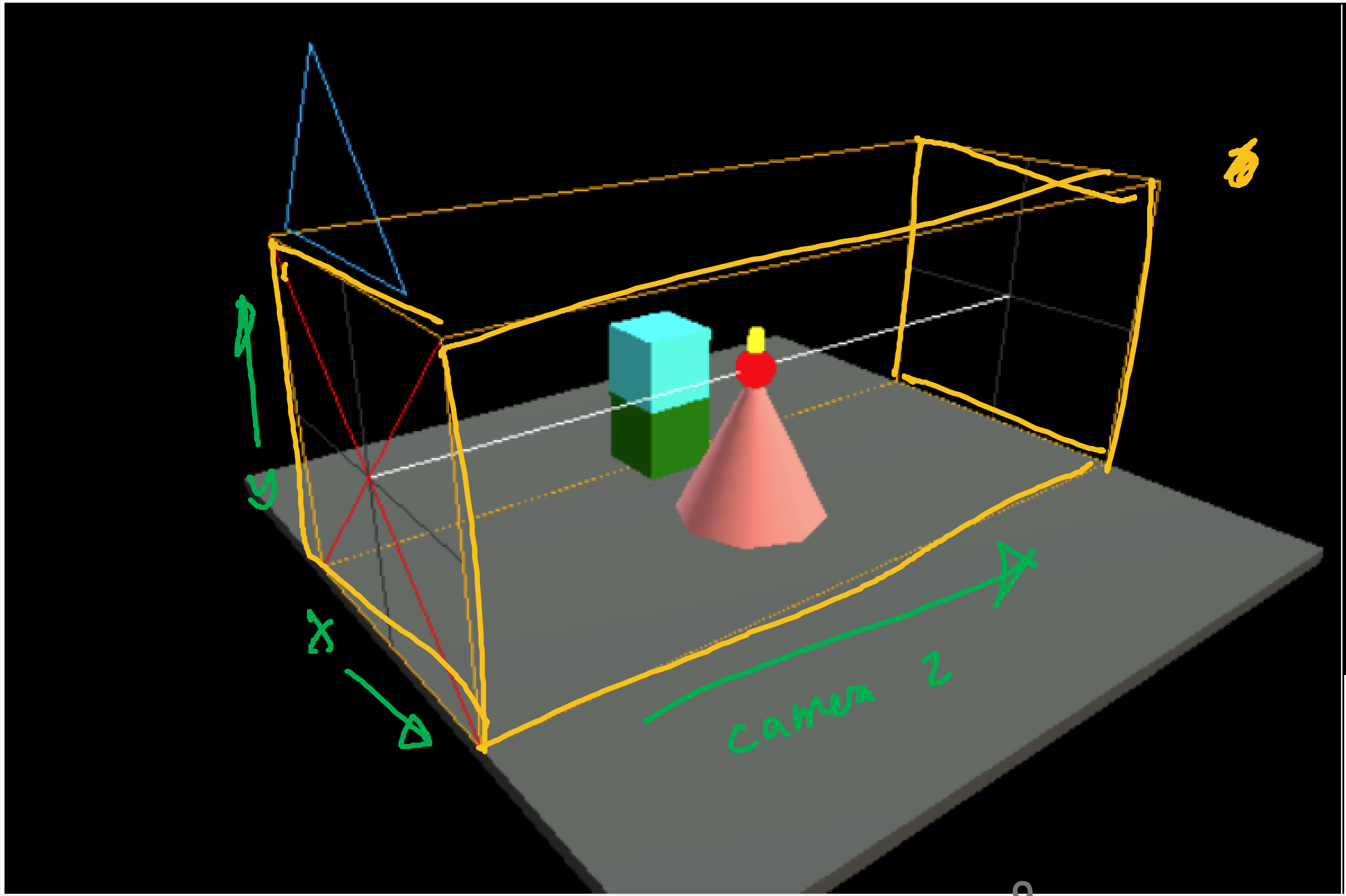
Orthographic Projection

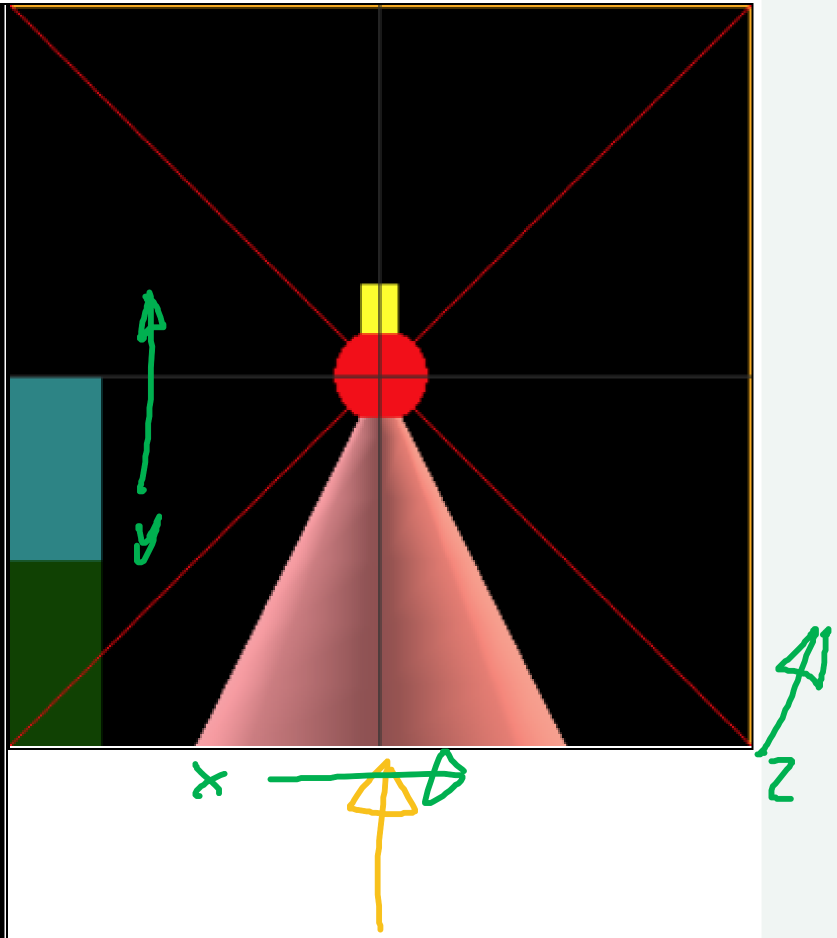
Projection = transformation that reduces dimension

Orthographic = flatten the world onto the film plane



The Orthographic "Box"



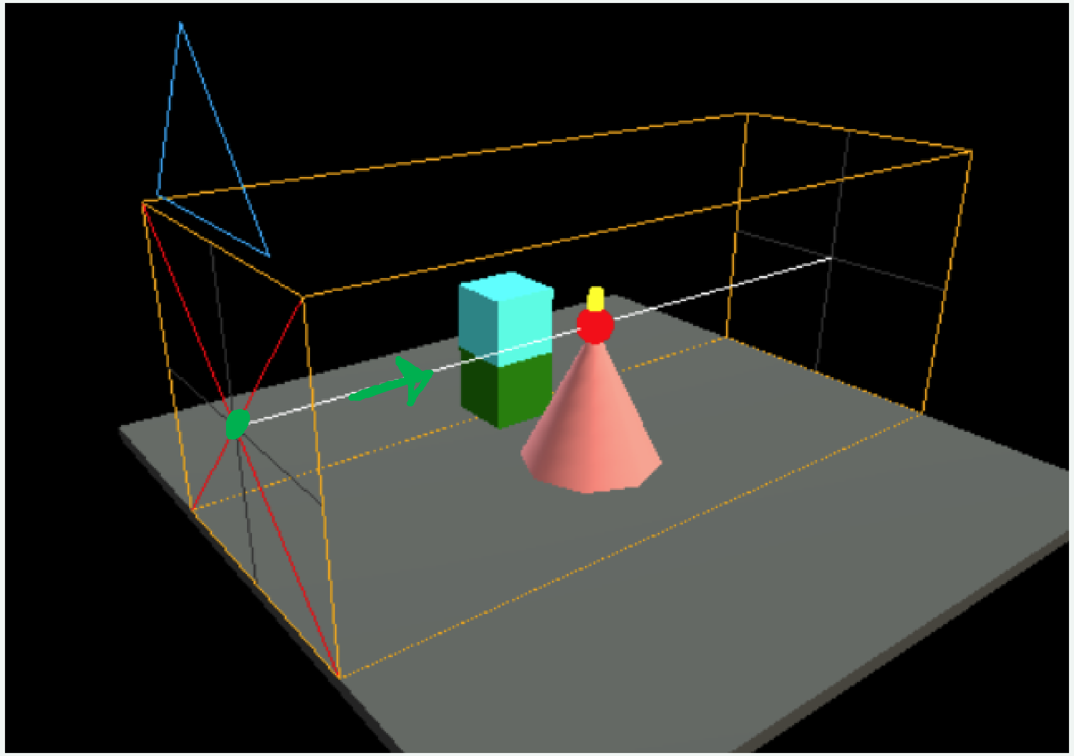


The Orthographic "Box"

It is a "Camera Object"

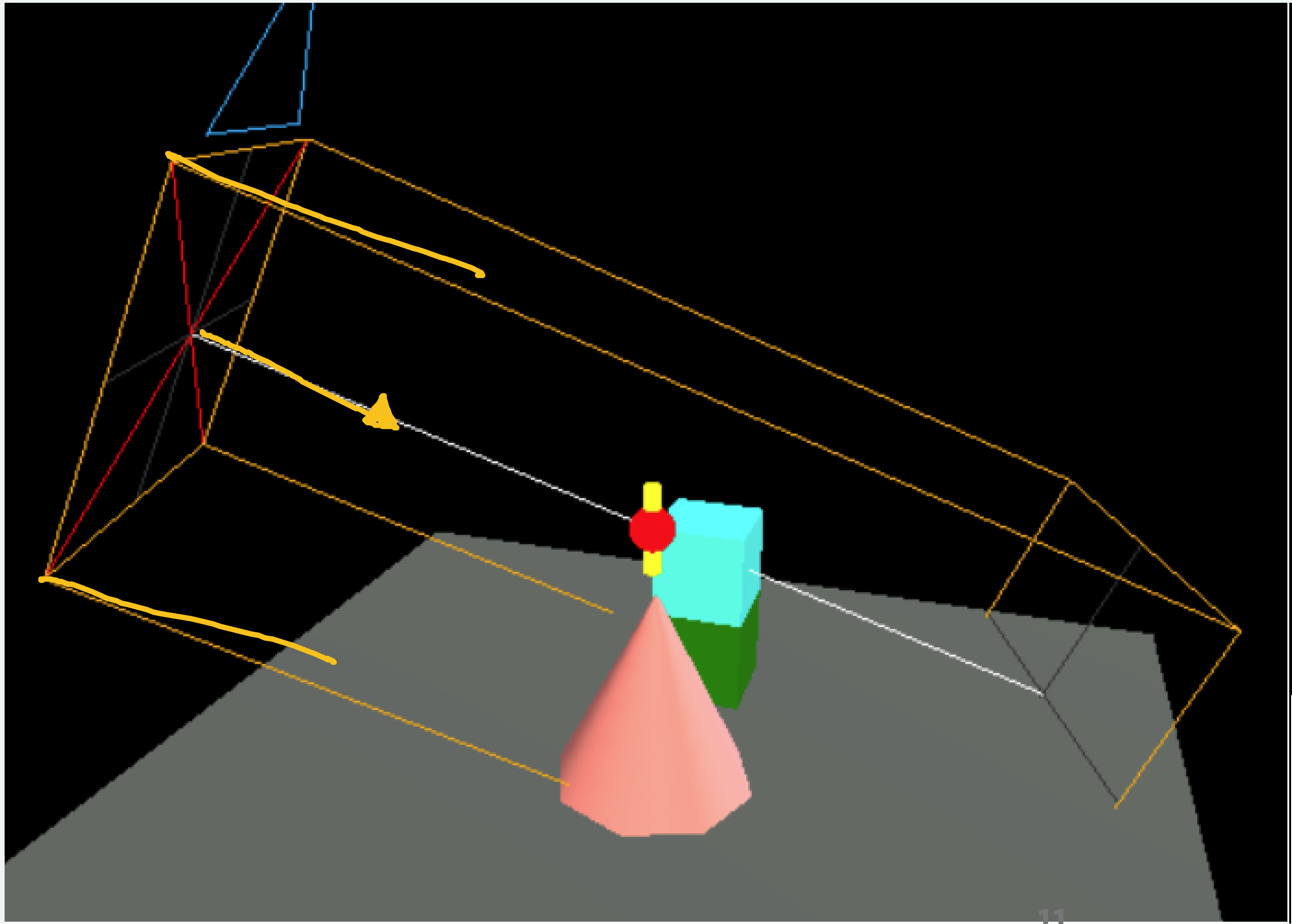
It is a Box in the World

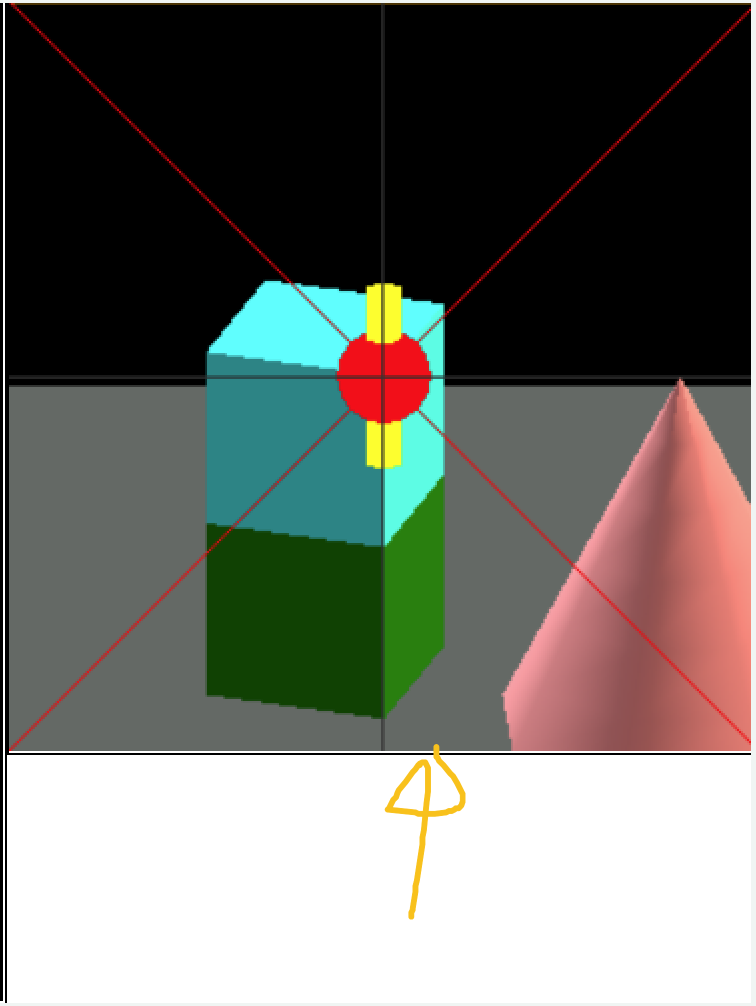
- position (eye point)
- forward direction (neg Z)
- up direction (Y)
- size (left/right/top/bottom)
- front/back



Box maps to "screen box"

You can orient the Box (rotate)





Orthographic

```
new T.OrthographicCamera(-2,2, -2,2, -2,2);
```

The screen (x,y,z)

Shift and scale to fit

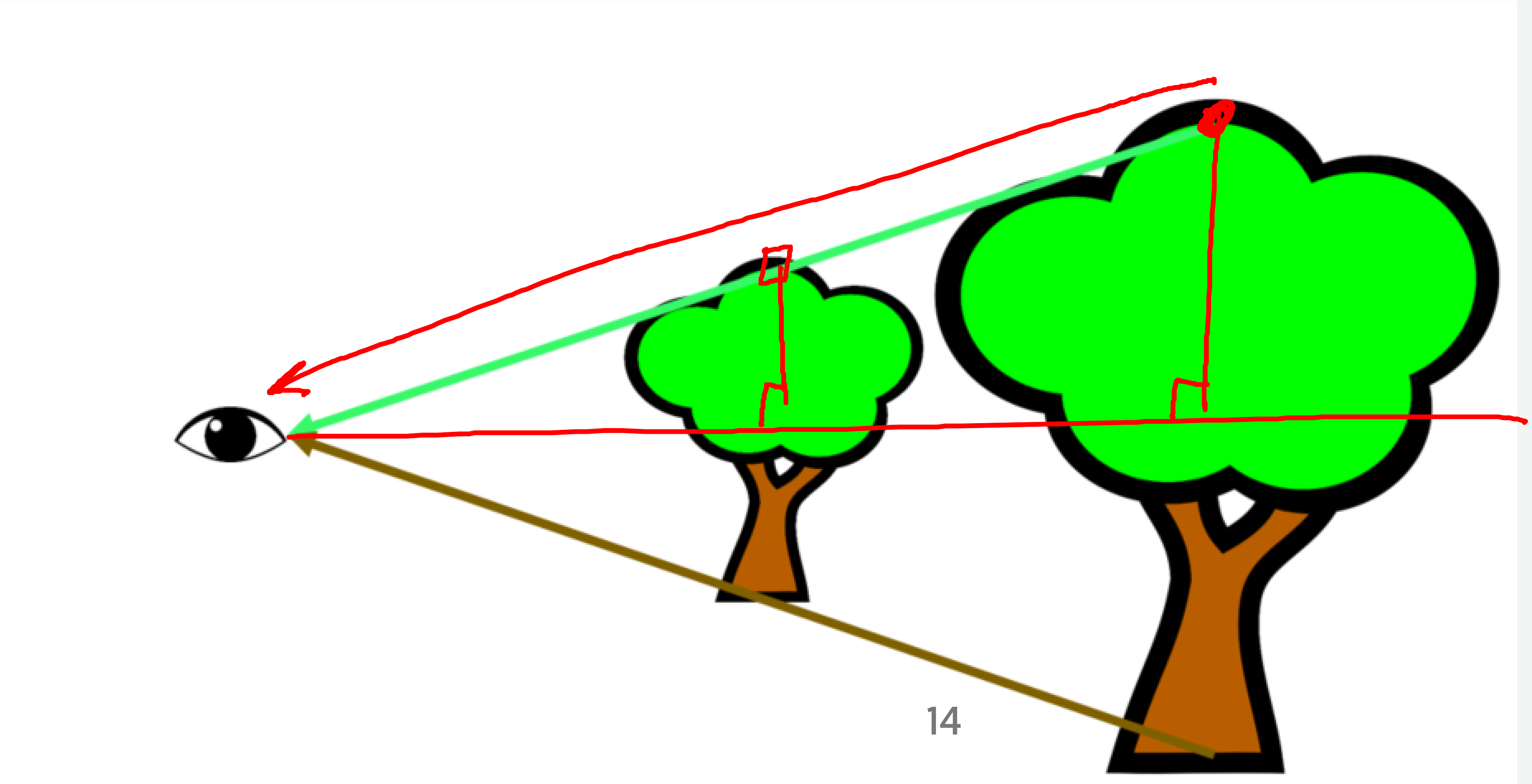
Rotations to get top, side, front

Scales the "box" to the "screen box"

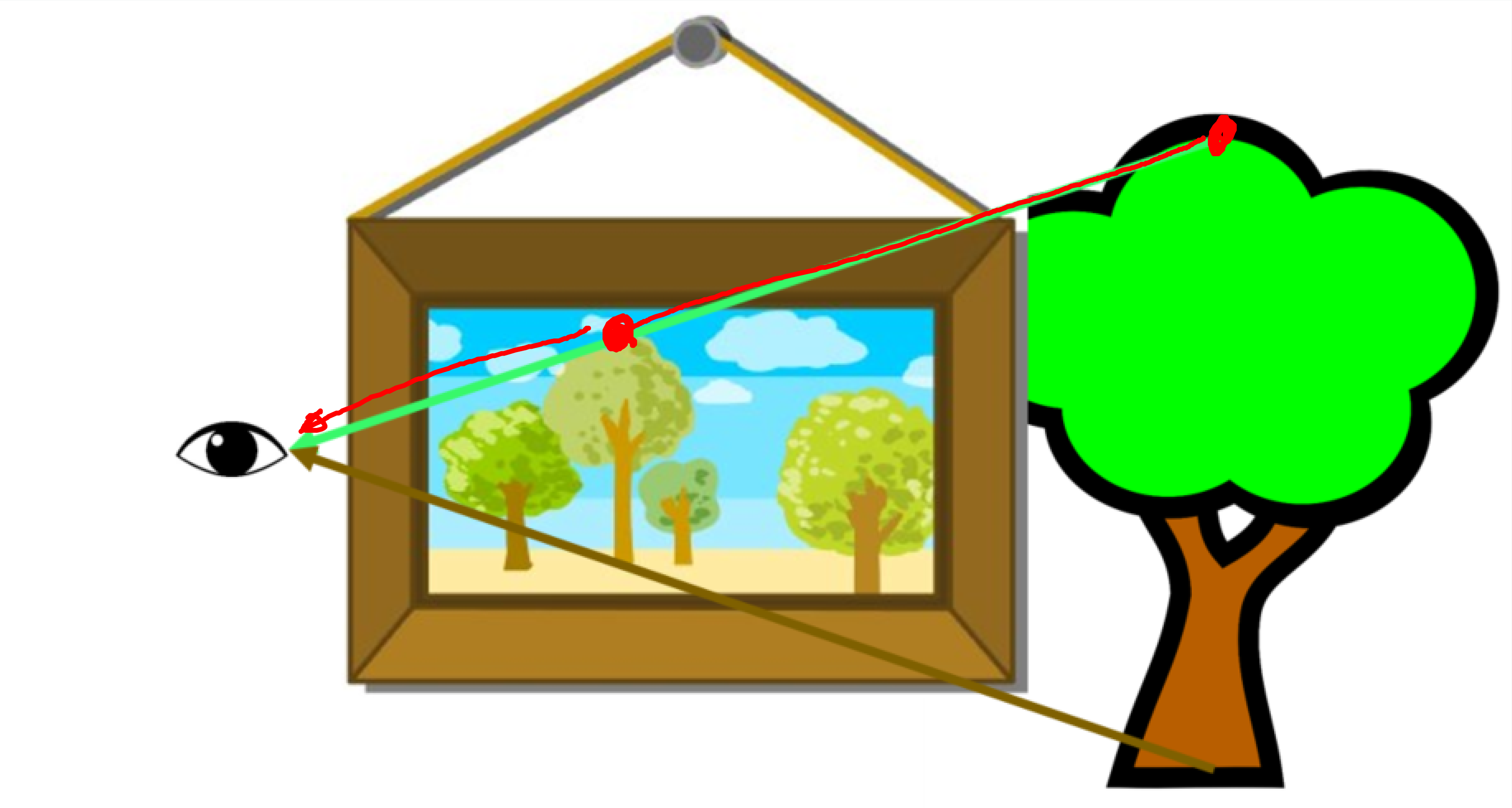
Perspective

Perspective

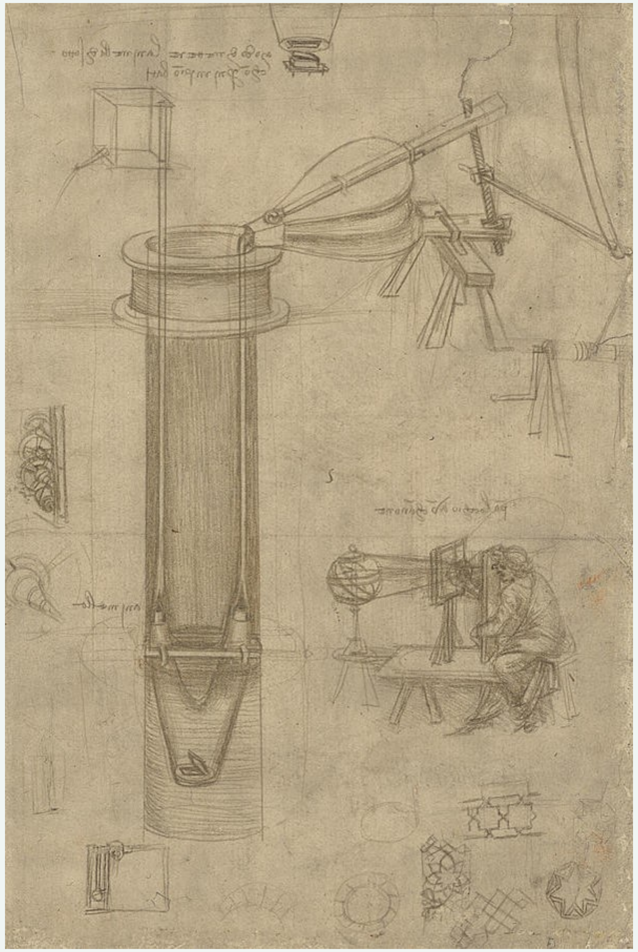
Things that are far look smaller

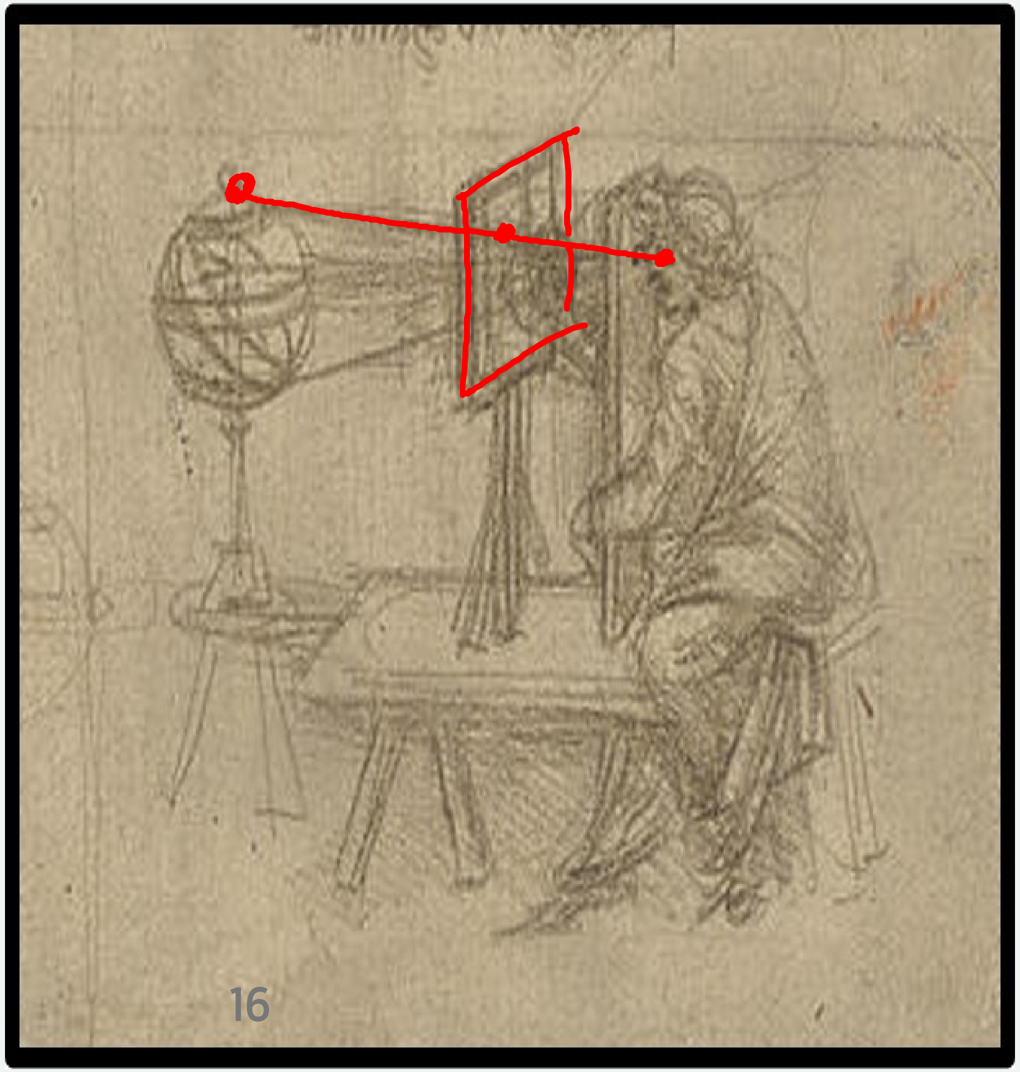


Looking at things: Depth and Distance

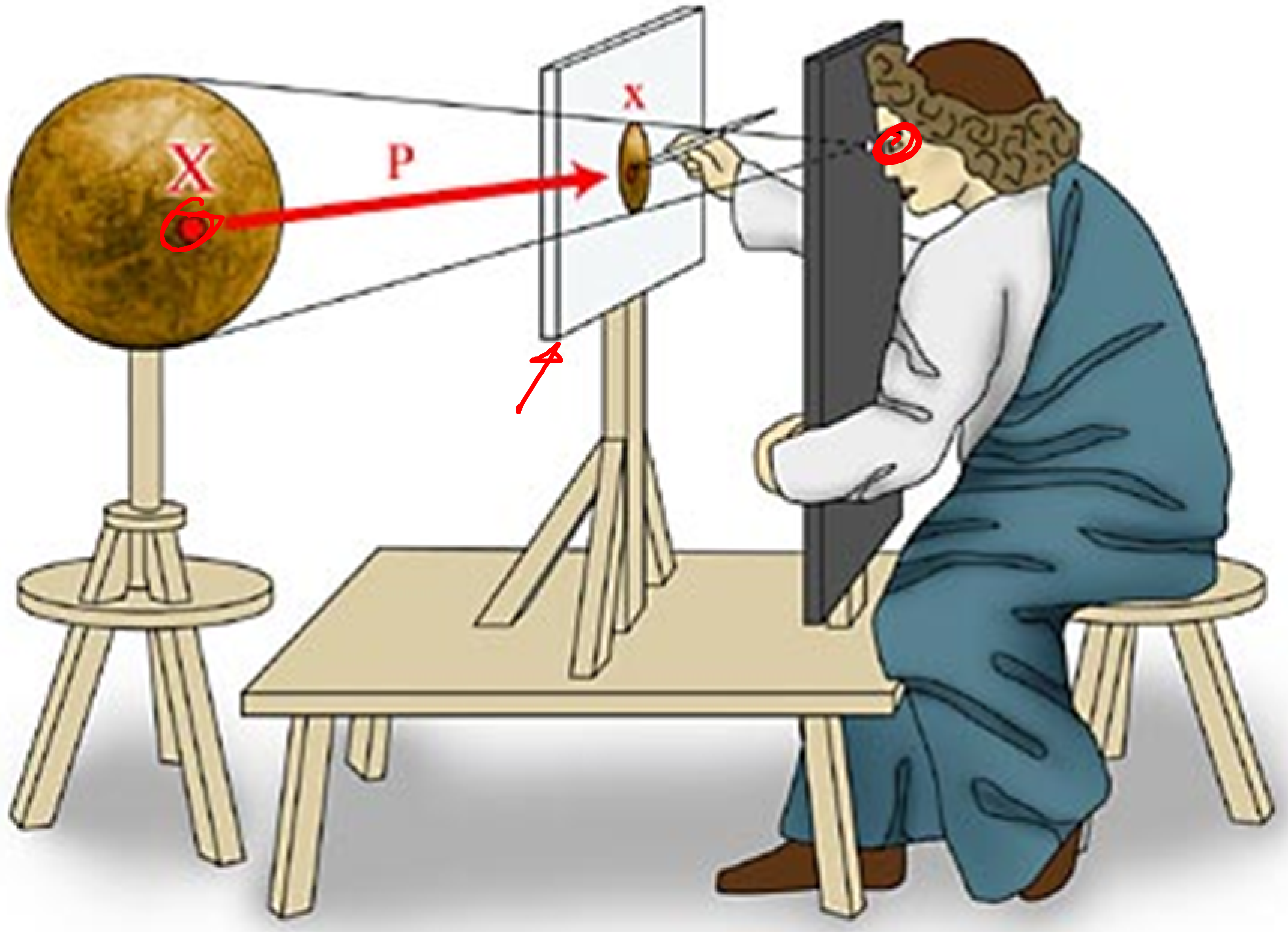


Do it like Da Vinci!





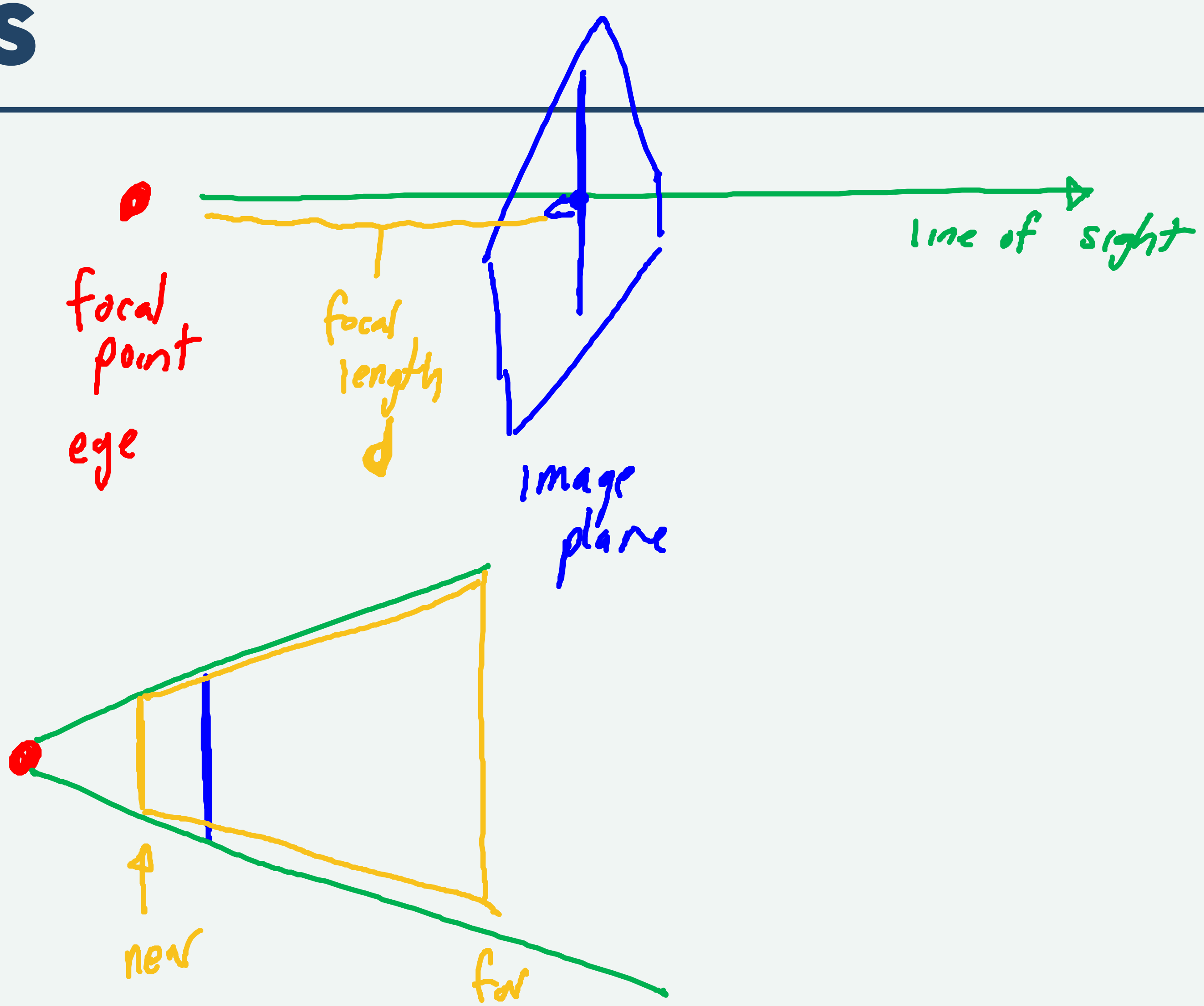
http://fineartamerica.com/featured/bellows-perspectograph-with-man-examining-inside-from-atlantic-codex-leonardo-da-



http://hans.wyrdweb.eu/about-perspective/

The intuitions

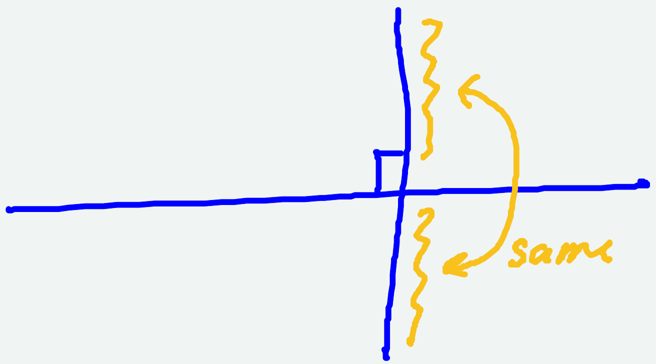
- focal point
- line of sight
- image plane
- focal length
- field of view
- frustum



Some Assumptions

Simple cameras: (general cameras can relax these)

- single focal point
- line of sight perpendicular to image plane
- line of sight centered

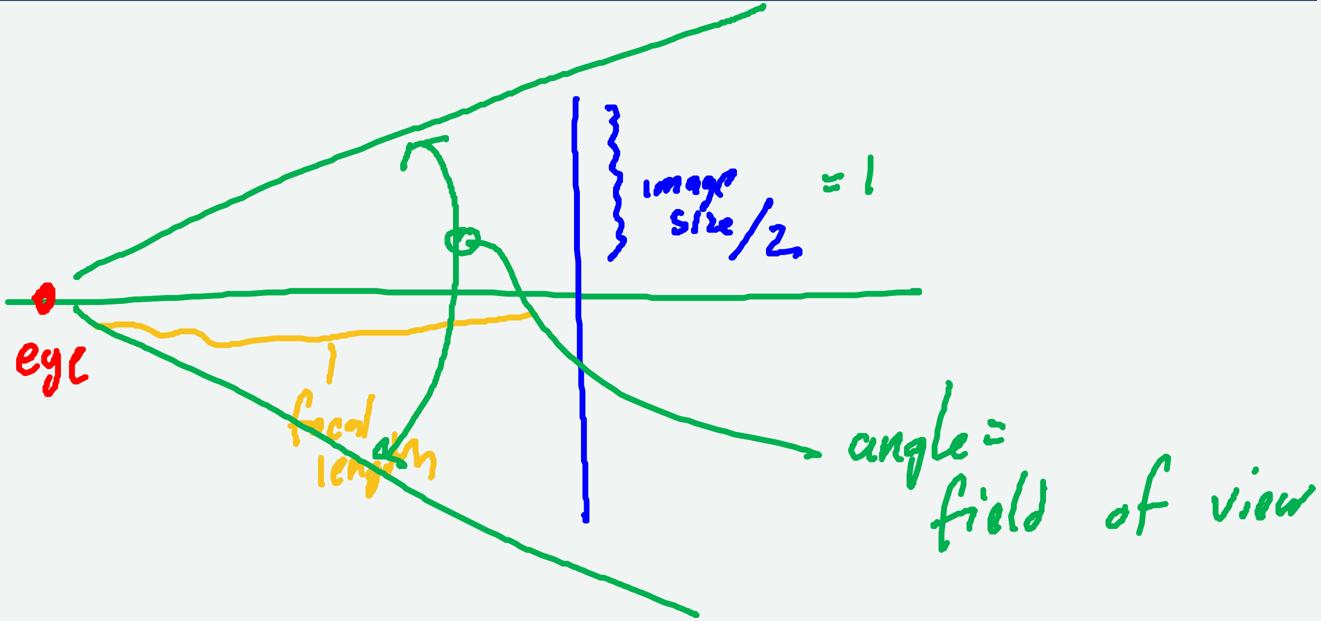


Simpler math

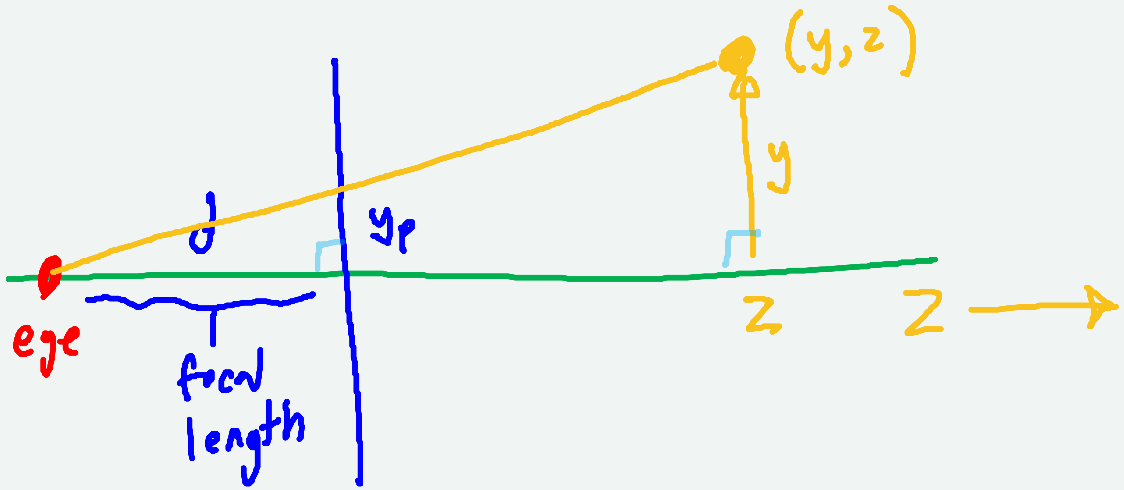
- sight down Z axis (or negative Z)

Field of View vs. Focal Length

- angle
- distance (film size)



Perspective Math: Similar Triangles



$$\frac{y}{z} = \frac{y_p}{d}$$

$$y_p = \frac{y\,d}{z}$$

The Math

$$x_s = \frac{d}{z}x \qquad y_s = \frac{d}{z}y \qquad z_s = d$$

This assumes that we are looking down the z axis

Linear?

$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad or \quad \begin{aligned} x_p &= d\,x \\ y_p &= d\,y \\ z_p &= 1 \\ w_p &= z \end{aligned}$$

Don't forget the divide by w!

Note what happens to z

$$X_s = \frac{X_p}{W_p} = \frac{dx}{z}$$

$$Z_s = \frac{Z_p}{W_p} = \frac{1}{z}$$

Z ordering

Cannot keep z!

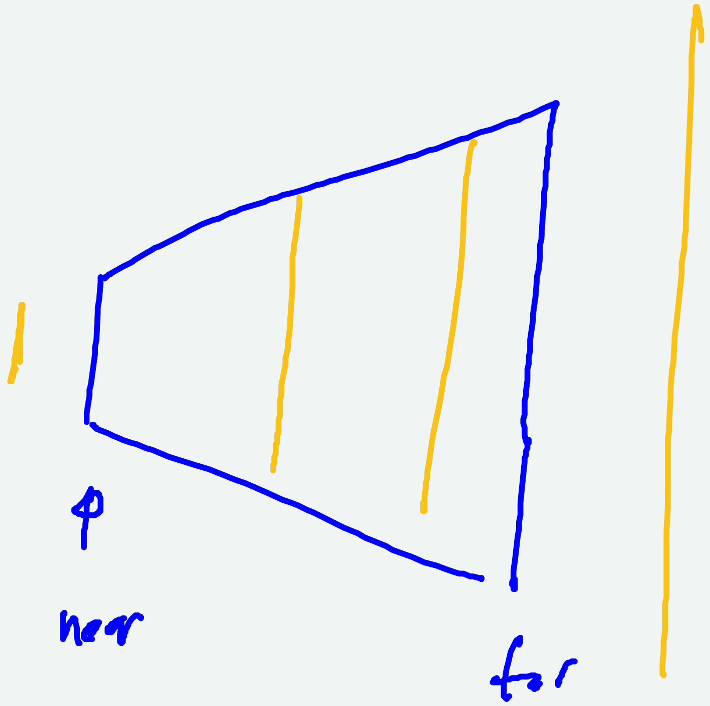
- if we divide by z, then the numerator would have to be z^2

We can get $\frac{1}{z}$

- preserves order (reverse - large Z is small value)
- breaks if $z = 0$, or negative
- shift z so it goes from near to far

Near and Far

Near is not the focal length



Is it really that simple?

Almost

A couple of catches:

- we need to scale z appropriately
- we need to scale x/y appropriately
- we're sighting down the positive/negative z
- the book discusses this well

The Matrix in the Book

$$\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

n - near plane distance

f - far plane distance

It's just a transformation!

Just like any other linear transformation

In THREE

```
let cam = new T.PerspectiveCamera(fov,aspect,near,far);
```

- `fov` is angle in degrees
- `aspect` is width/height (needs to match canvas)
- `near` - anything closer is not seen
- `far` - anything farther is not seen

This is an Object3D.

It isn't visible, but it has all the transformations.

Summary

1. Projections and Fishtank Model
2. Near and Far
3. Orthographic Box
4. Perspective Concepts
5. Simple Perspective Model
6. Perspective Math (simple and in practice)
7. Persepctive in THREE