Lecture 26[redacted]
Free-Form Surfaces
Subdivision

Shape

- Curves vs. Surfaces vs. Solids
- Surface "primitives" (spheres, cylinders, cones, ...)
- Surface "general primitives" (generalized cylinders, cones, sweeps, lofts)
- Mesh
- Free form surfaces

Free Form Surfaces: Approaches

Same as curves

- Parametric: $(x, y, z) = \mathbf{f}(u, v)$
- Implicit: $f(x, y, z) = 0$
- Procedural
- Subdivision

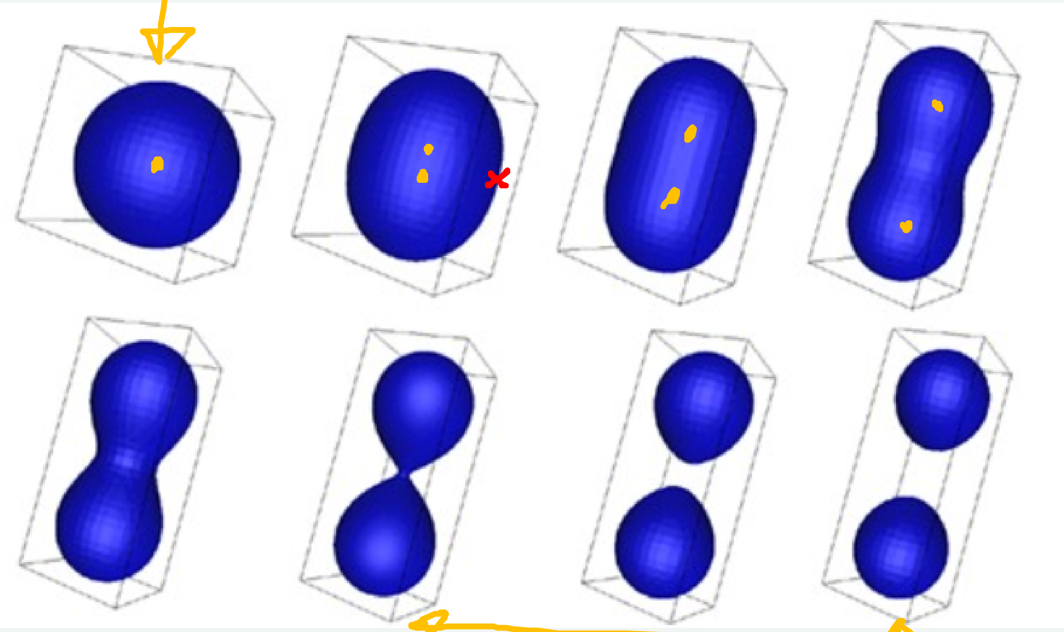
With curves, parametric is the common choice.

With surfaces, parametric is problematic so the others can be worthwhile.

Implicit Surfaces

$f(x, y, z) = 0$

- inside of sphere
- inside of set of spheres
- distance to a set of points
- density (blobs)
  - (falls of to zero quickly)
- model by summing blobs

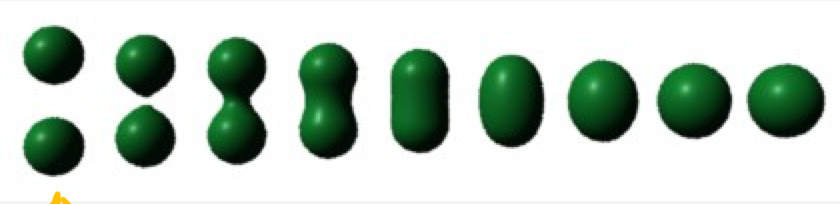


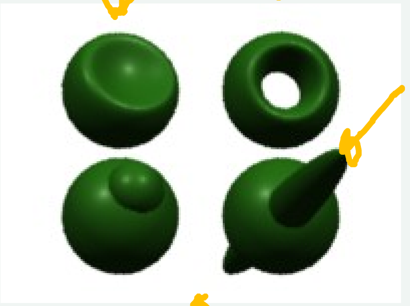
$x^2 + y^2 + z^2 - r^2 = 0$

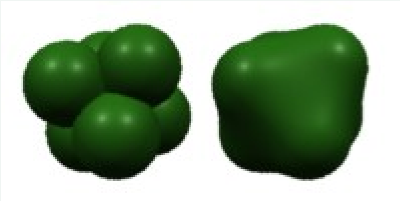
2D circle - $x^2 + y^2 - r^2 = 0$

Why do we like this?

Easy to combine simple units







How to draw an implicit surface?

Need to find points on $f(x, y, z) = 0$

Parametric

Free form surfaces

Is there an analog to polynomial curves?

$$f(u) \to \mathcal{R}^3$$

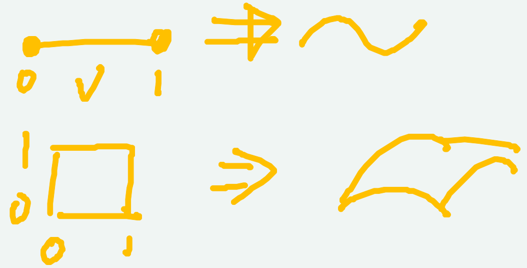
free parameter

parametric function

Cubic Polynomials

curve: $f(u) = a_0 + a_1 u^1 + a_2 u^2 + a_3 u^3$

surface: $f(u, v) = ???$



Polynomial in u and v! (tensor product)

$$
\begin{aligned}
f(u, v) = & a_{00} u^0 v^0 + a_{01} u^1 v^0 + a_{02} u^2 v^0 + a_{03} u^3 v^0 + \\
& a_{10} u^0 v^1 + a_{11} u^1 v^1 + a_{12} u^2 v^1 + a_{13} u^3 v^1 + \\
& a_{20} u^0 v^2 + a_{21} u^1 v^2 + a_{22} u^2 v^2 + a_{33} u^3 v^2 + \\
& a_{30} u^0 v^3 + a_{31} u^1 v^3 + a_{22} u^2 v^3 + a_{33} u^3 v^3
\end{aligned}
$$

cubic polynomial patch

Tensor Product Surface Patches

16 coefficients (control points)!

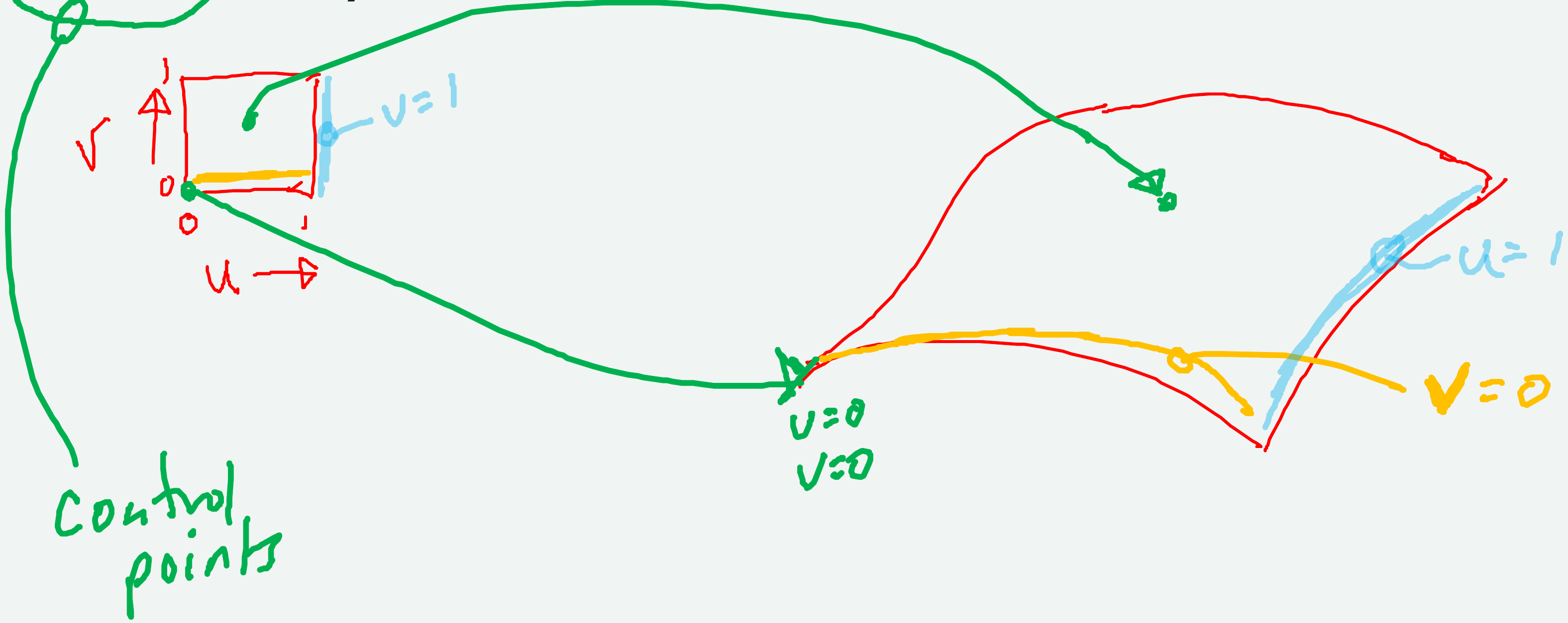
$$
\begin{aligned}
f(u,v) = a_{00}u^0v^0 &+ a_{01}u^1v^0 + a_{02}u^2v^0 + a_{03}u^3v^0 + \\
a_{10}u^0v^1 &+ a_{11}u^1v^1 + a_{12}u^2v^1 + a_{13}u^3v^1 + \\
a_{20}u^0v^2 &+ a_{21}u^1v^2 + a_{22}u^2v^2 + a_{33}u^3v^2 + \\
a_{30}u^0v^3 &+ a_{31}u^1v^3 + a_{22}u^2v^3 + a_{33}u^3v^3
\end{aligned}
$$

There are analogs to curve formulations

- Bezier, B-Spline, Interpolating, ...

Thinking about tensor product patches

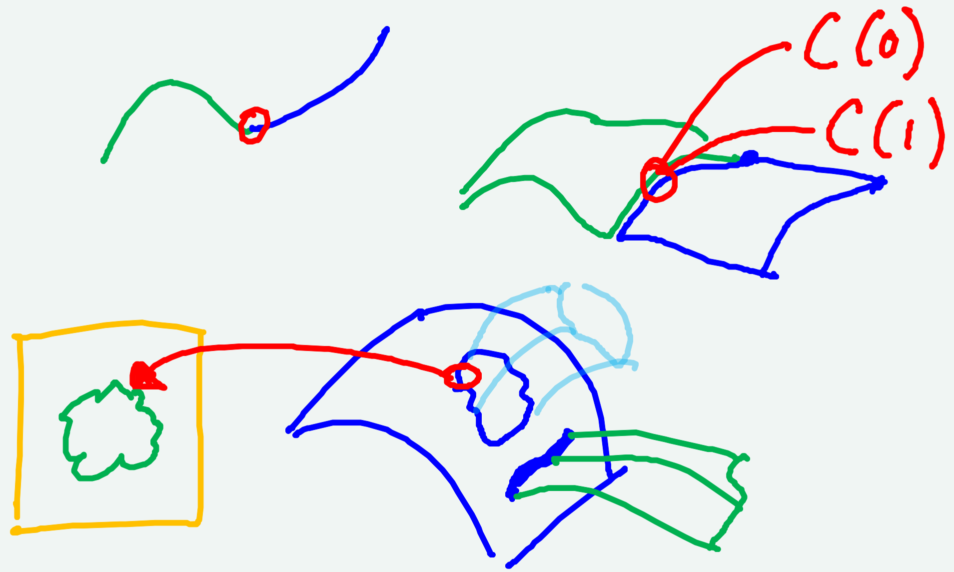
f(u,v) -> x,y,z (u,v in 0,1)



Tensor Product Surfaces are Hard!

How to connect two patches?

- Continuity
- Stitching together



How to cut a patch?

- Make a Hole? trimming
- Make an edge? (attachment)

How about non-square domains?

- inconvenient stretching?
- different topology?

What do we do instead?

Subdivision: Motivation

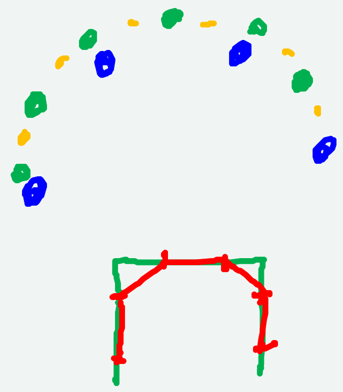
Polynomial Surfaces Are Challenging

f(u,v) -> x,y,z

- What if the patches aren't square?
- How do we connect them? (for smoothness)
- How do we cut holes in them?
- How do we stitch them together?

Subdivision: Intuitions from 2D

- Start with a set of (points) line segments
- Add new points / move old points
- Divide segments into more segments
- Repeat
  - until good enough
  - infinitely many times

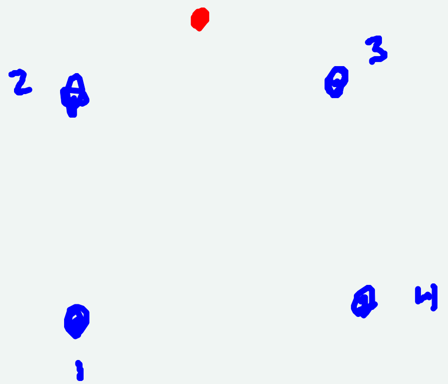


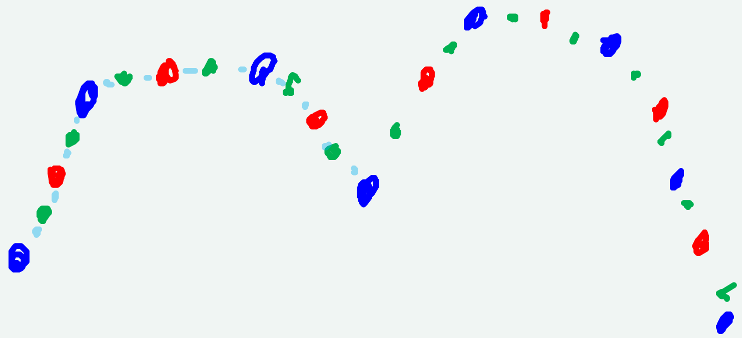
Design so it converges to a smooth curve

Example 1: Dyn/Levin/Gregory

4 point scheme - each new point looks at 4 neighbors

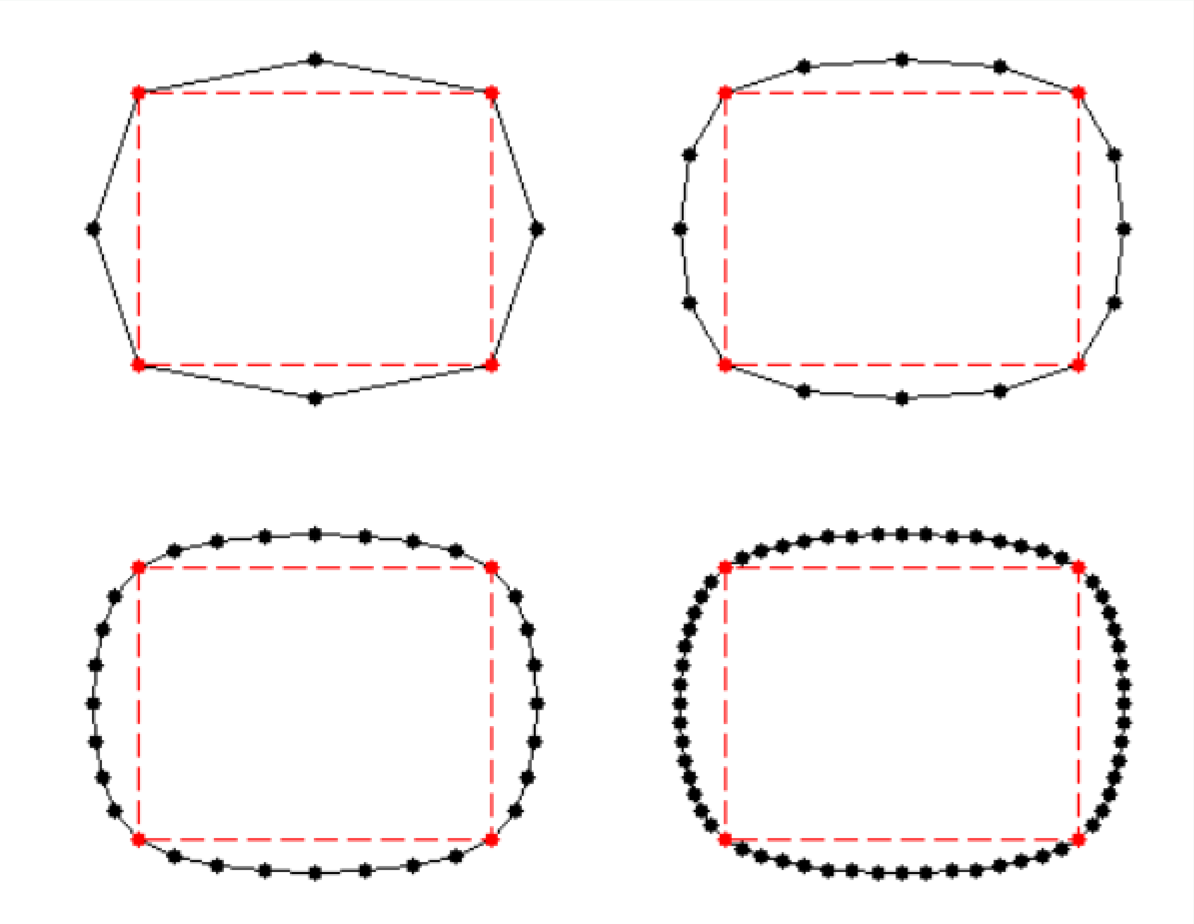
$$\left[\, -\frac{1}{16},\quad \frac{1}{2}+\frac{1}{16},\quad \frac{1}{2}+\frac{1}{16},\quad -\frac{1}{16} \,\right]$$





more generally $\left[\, -w,\quad \frac{1}{2}+w,\quad \frac{1}{2}+w,\quad \cancel{-\frac{1}{16}} - w \,\right]$

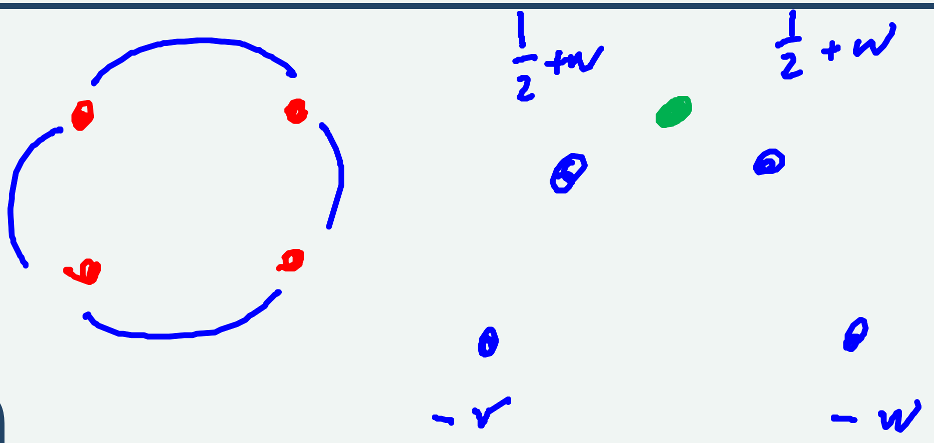
Each time it gets smoother…



Infinitely many times?

Converges to a cubic spline!

(you can read the proof)



Note: Interpolation

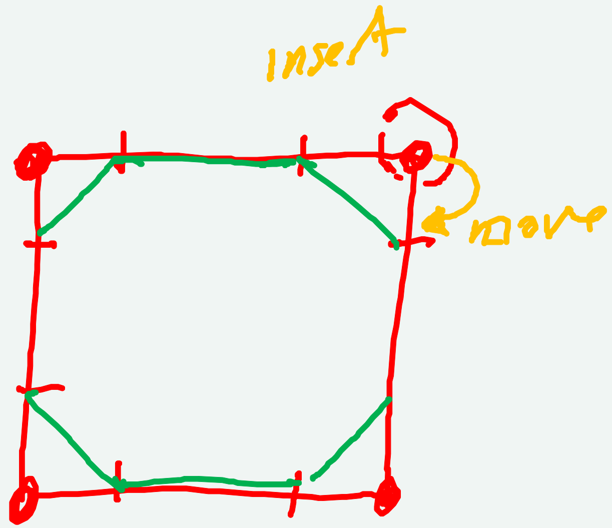
Original points continue - forever

Example 2: Not interpolating

Chakin Corner Cutting (from lecture 11)

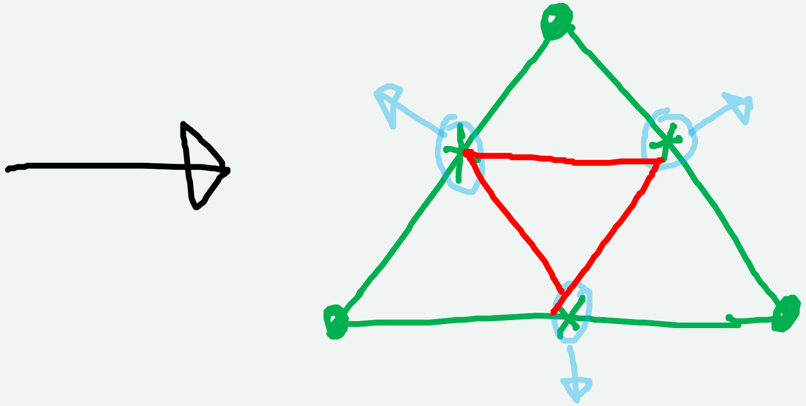
- each corner -> 2 points (1/4 from edge)
- each segment cut at (1/4, 3/4)

Converges to quadratic B-Spline



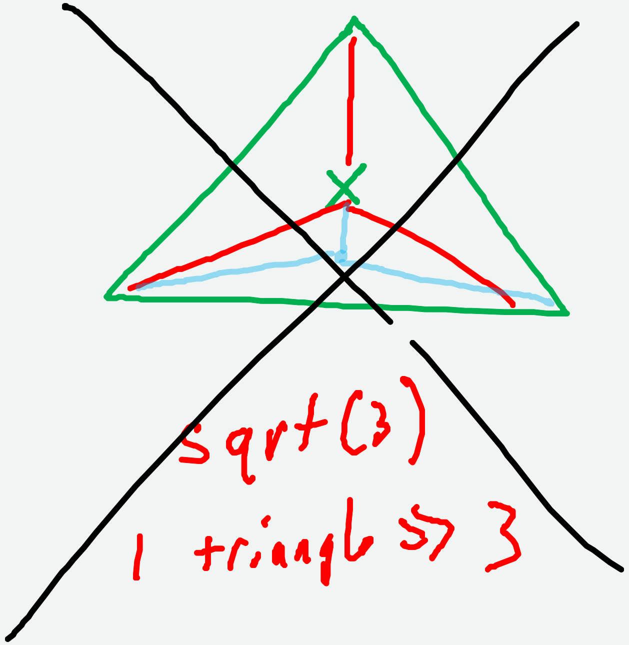
In 3D

- Cut each triangle into new triangles
  - place the new vertices
  - move the old vertices (non-interpolating)



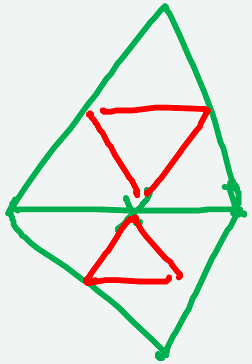
split edges

1 triangle ⇒ 4



sqrt(3)

1 triangle ⇒ 3

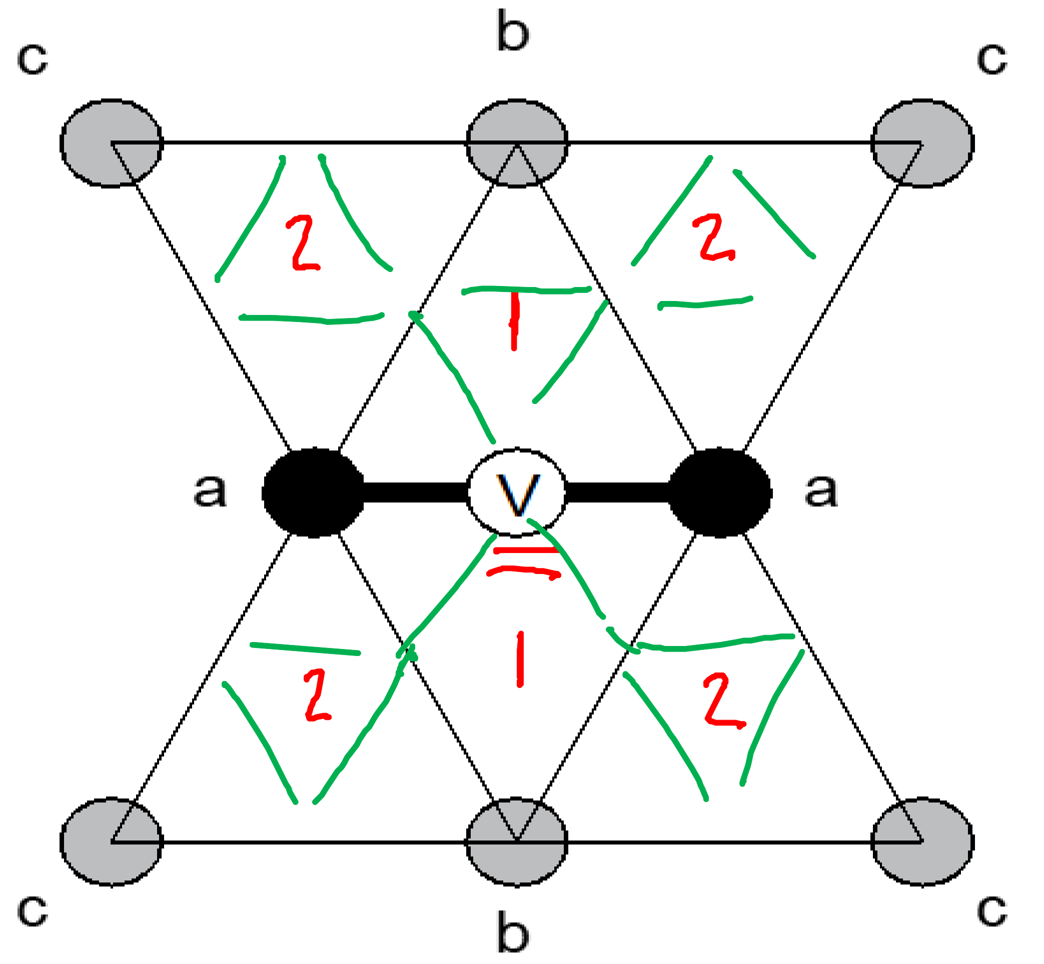


Dividing triangles

Standard (4-way) scheme

~~3-way scheme~~

Butterfly

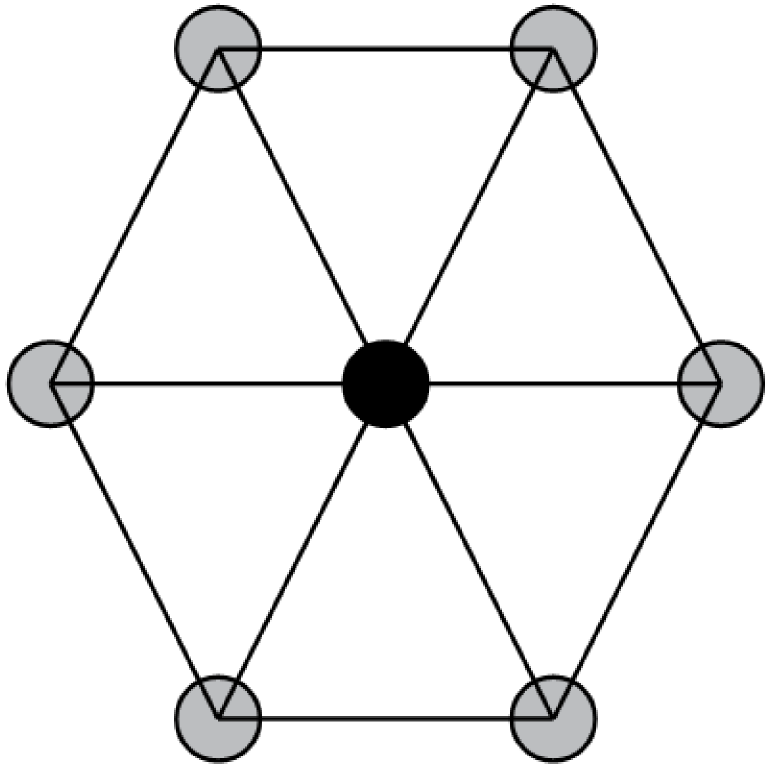


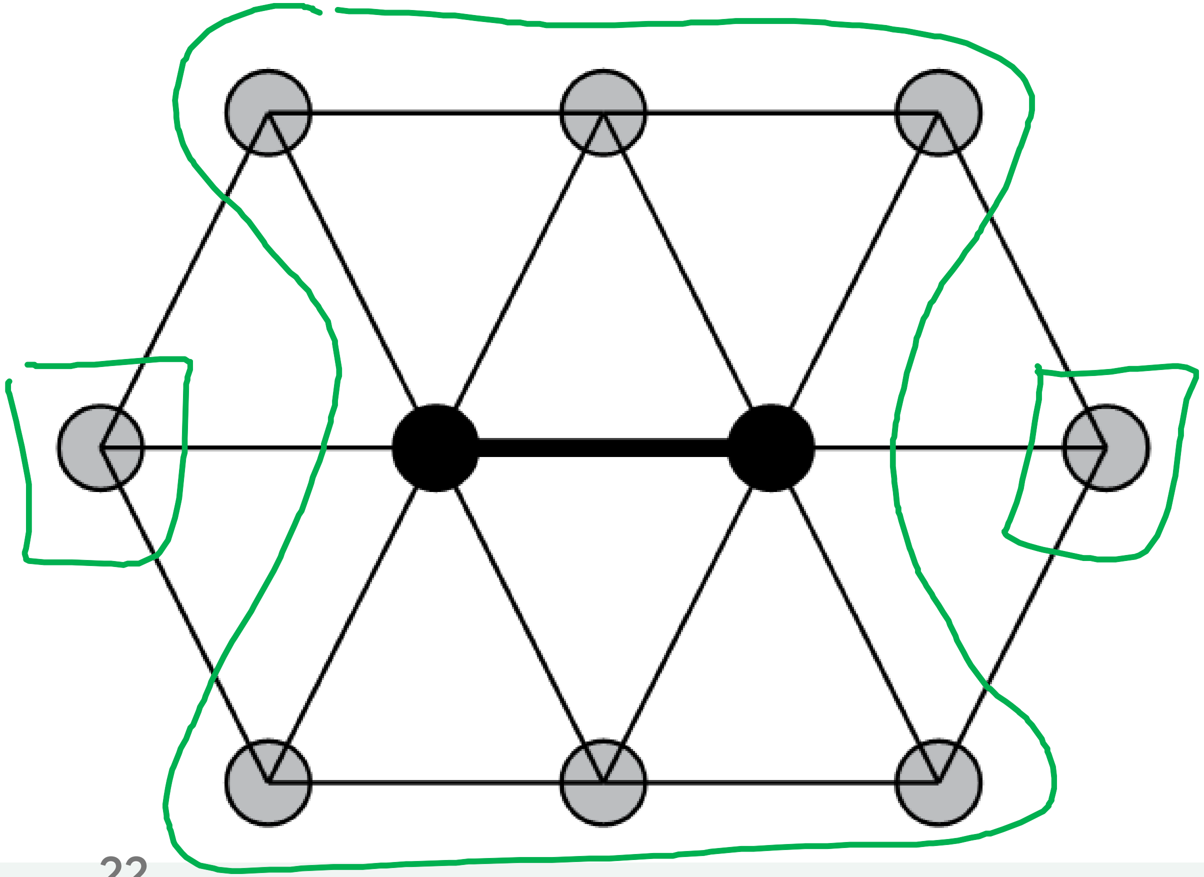
$$v = 1/2\, a + 1/8\, b - 1/16\, c$$

2 2 4

Uniform Meshes

Ordinary and Extra-Ordinary Points

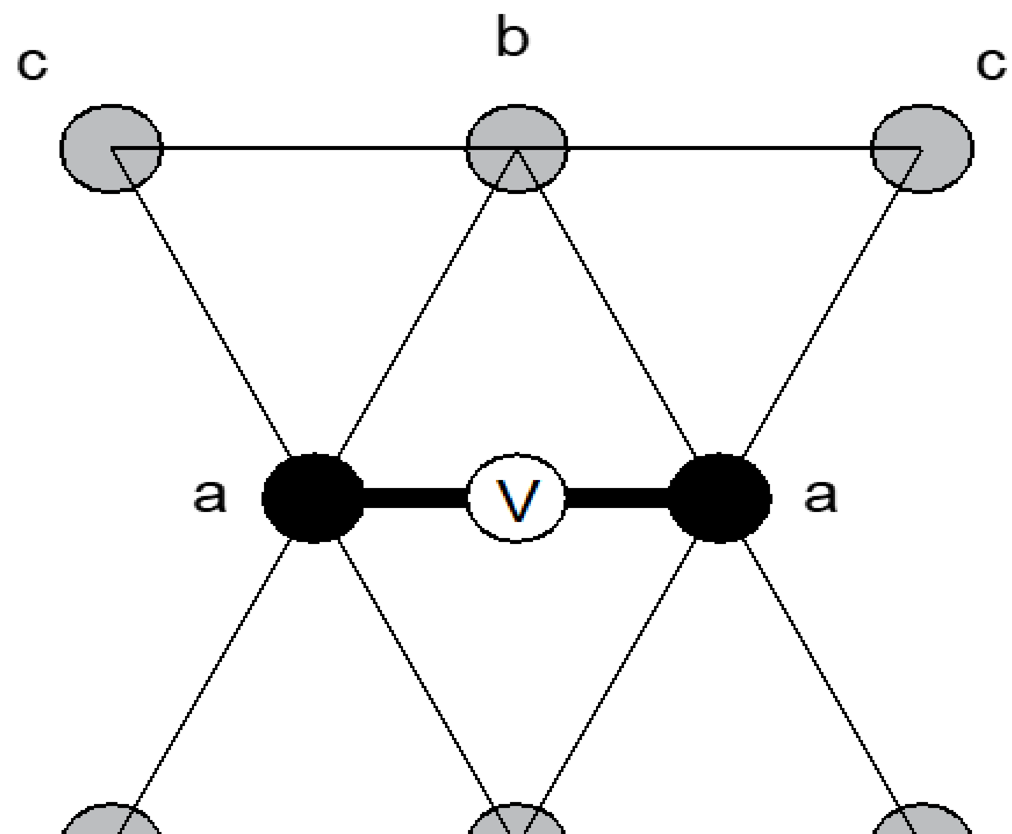




Butterfly

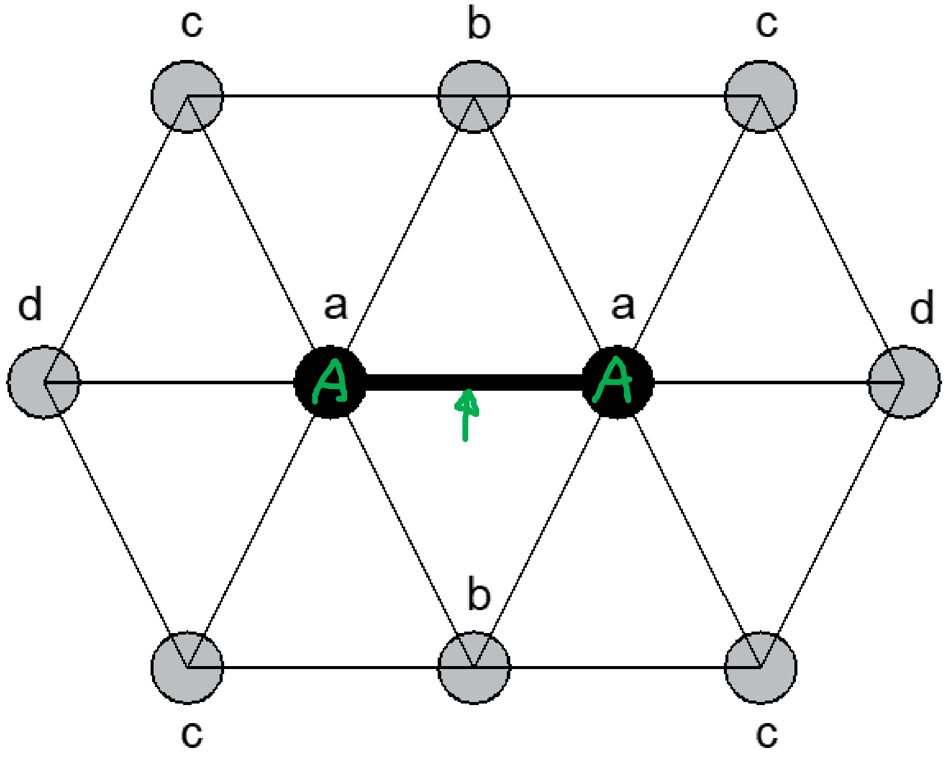
C(1) almost everywhere

Special rules for extra-ordinary points



$$v = 1/2\,a + 1/8\,b - 1/16\,c$$

Modified Butterfly

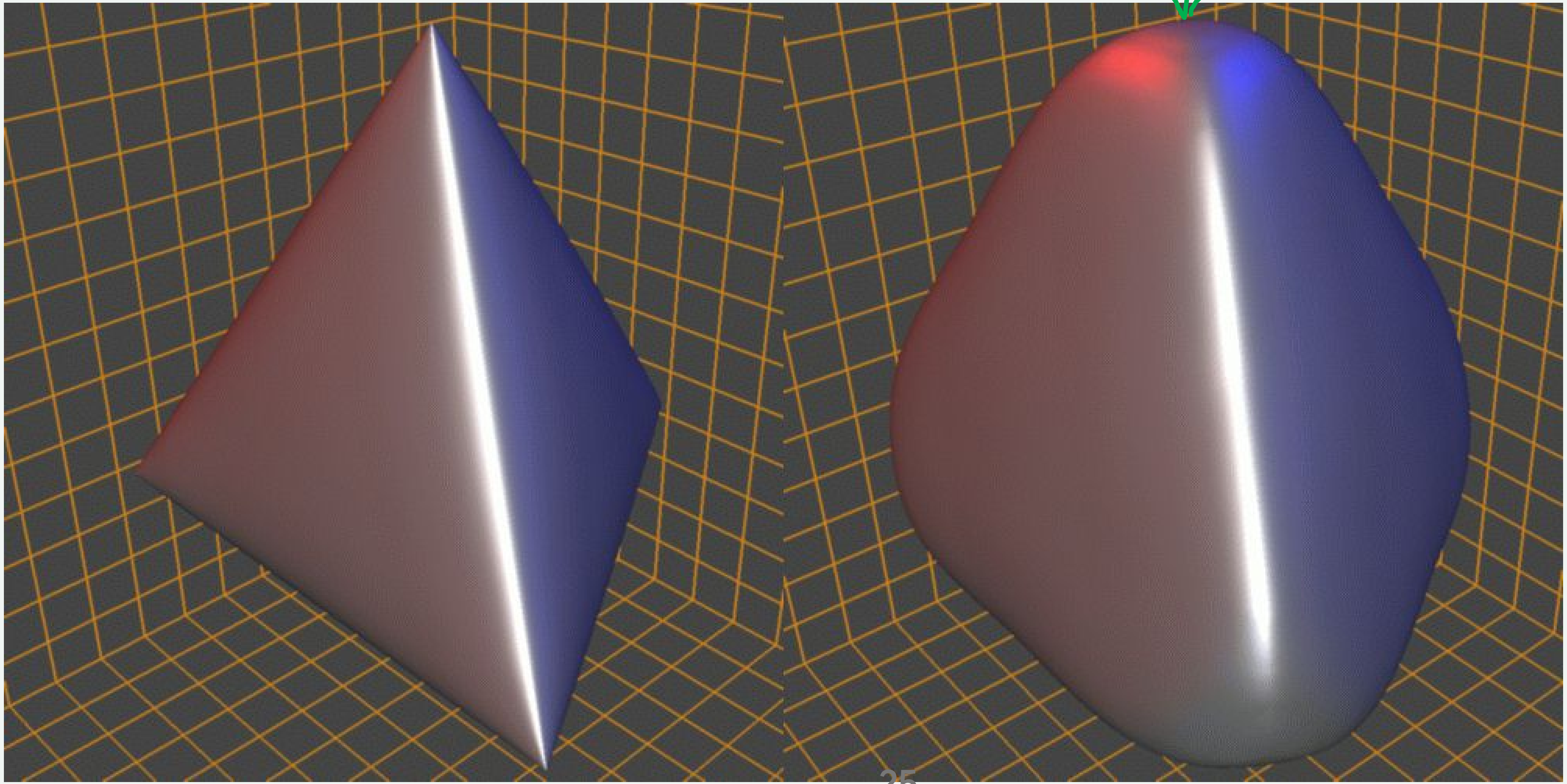


$$\mathbf{v} = (1/2-w)\,\mathbf{a} + (1/8+2w)\,\mathbf{b} - (1/16-w)\,\mathbf{c} + w\,\mathbf{d}$$

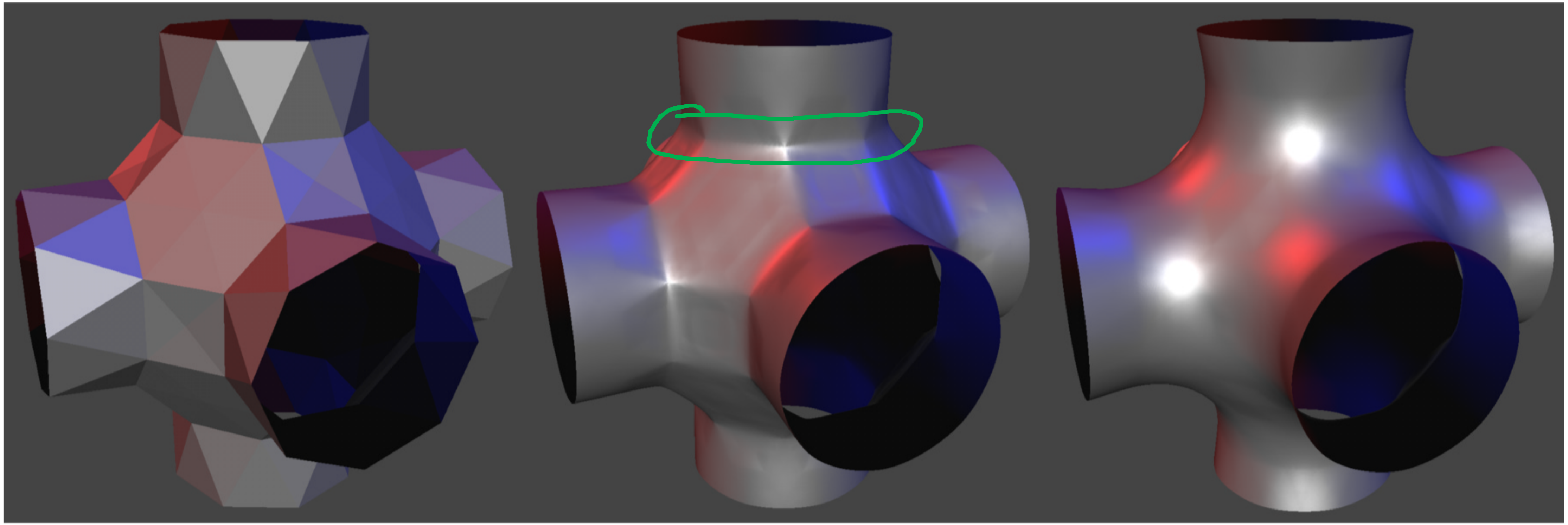
tension parameter w
sum over all 10 neighbors

Tension

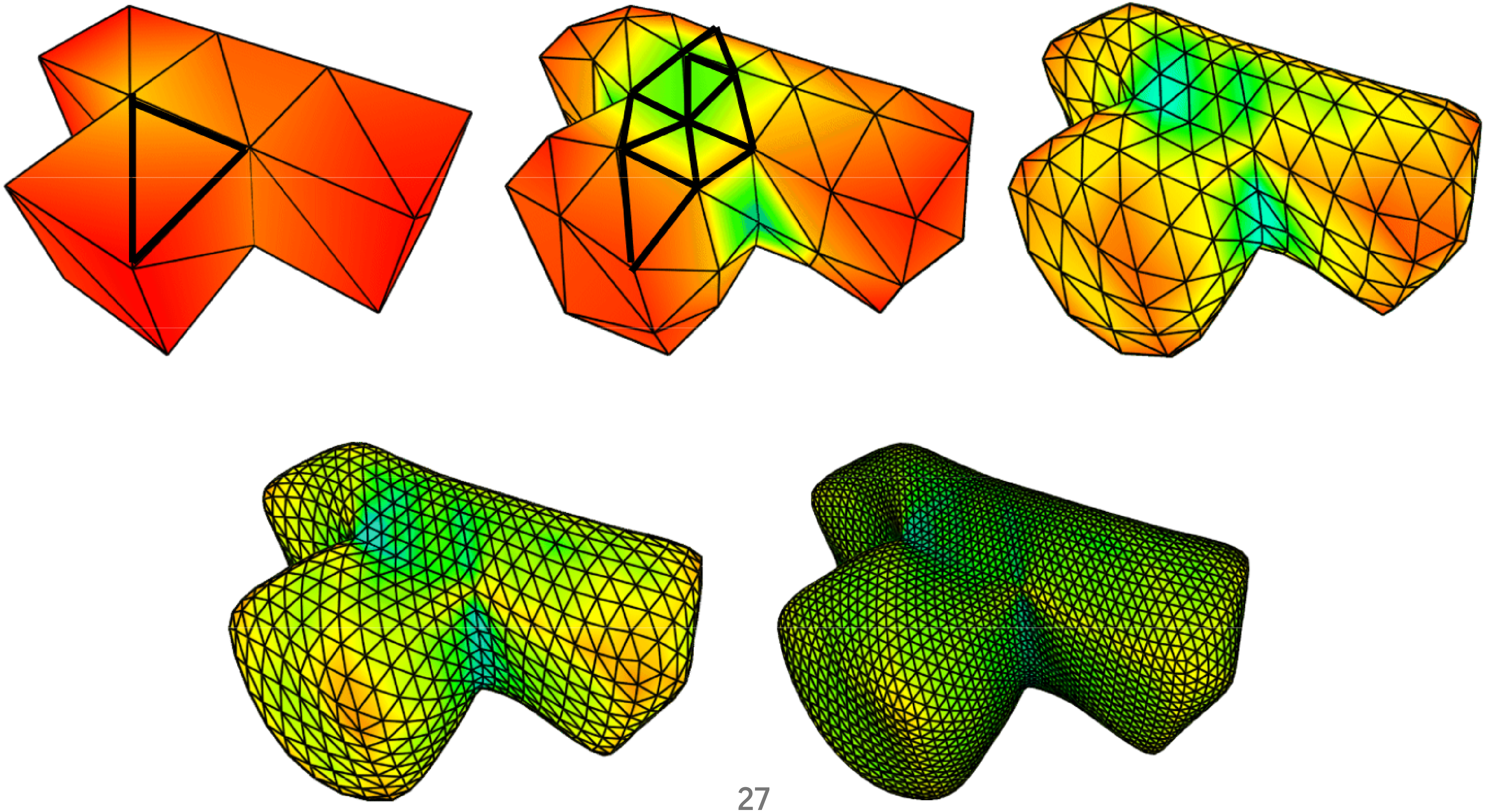
$w = 0$



Butterfly vs. Modified



Initial mesh | Butterfly scheme interpolation | Modified Butterfly interpolation



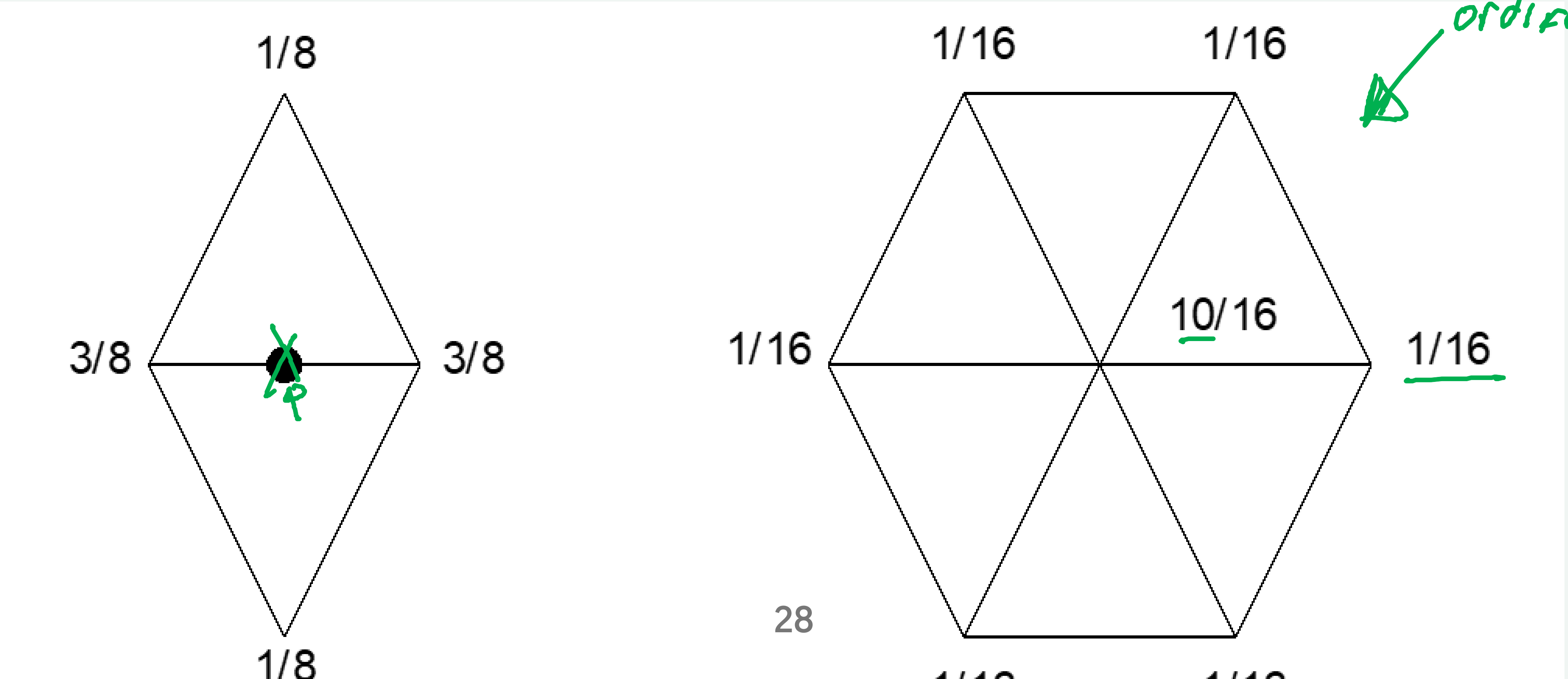
http://graphics.stanford.edu/courses/cs468-10-

Loop Scheme

Charles Loop

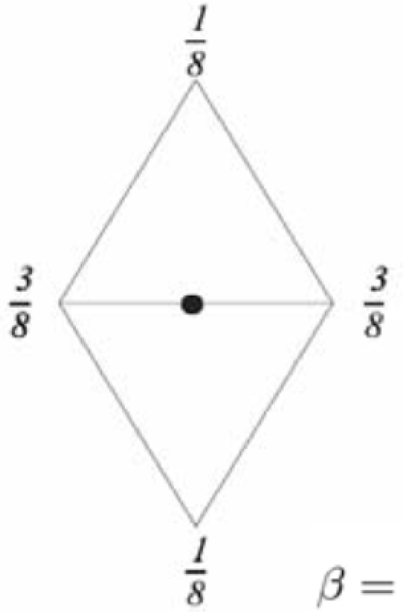
- New points split edges
- Old points moved to smooth ← not interpolating

ordinary

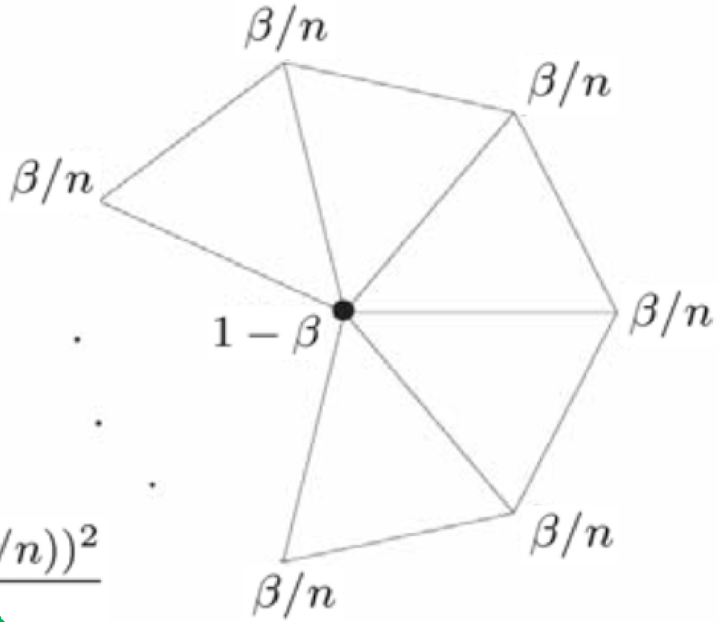


Loop Rules - General (irregular)

Full Loop rules (triangle mesh)



Interior

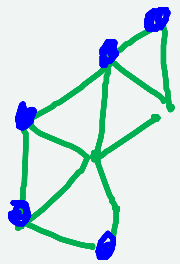


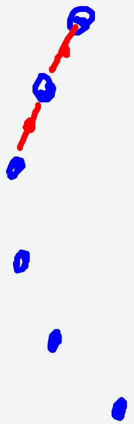
$$\beta = \frac{5}{8} - \frac{(3 + 2\cos(2\pi/n))^2}{64}$$

Loop Rules - Boundaries

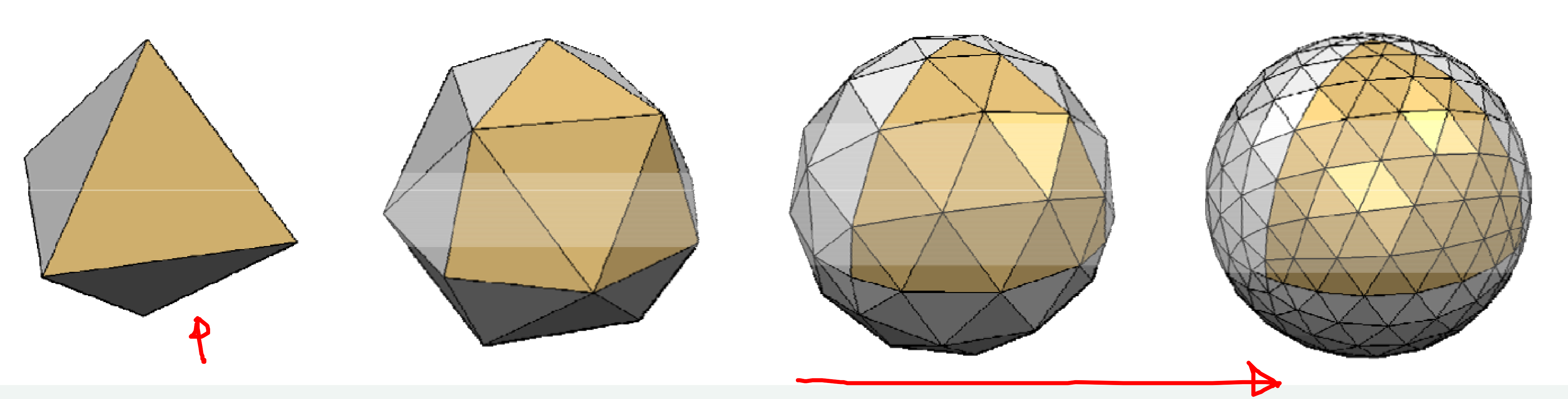
- new points half way
- old points 1/8 3/4 1/8
- edges only depend on edges

1/8 3/4 1/8





Loop Example

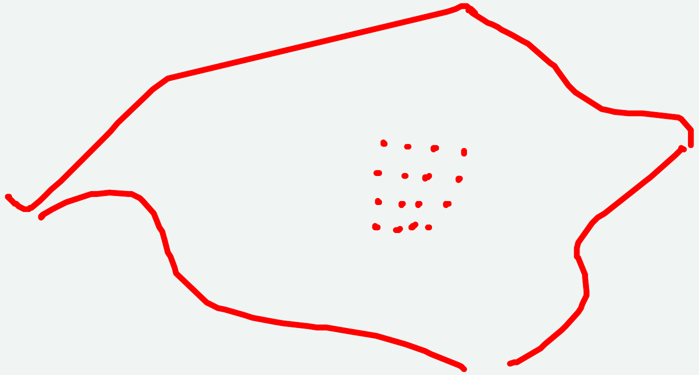


http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/10_Subdivision.pdf

In the limit?

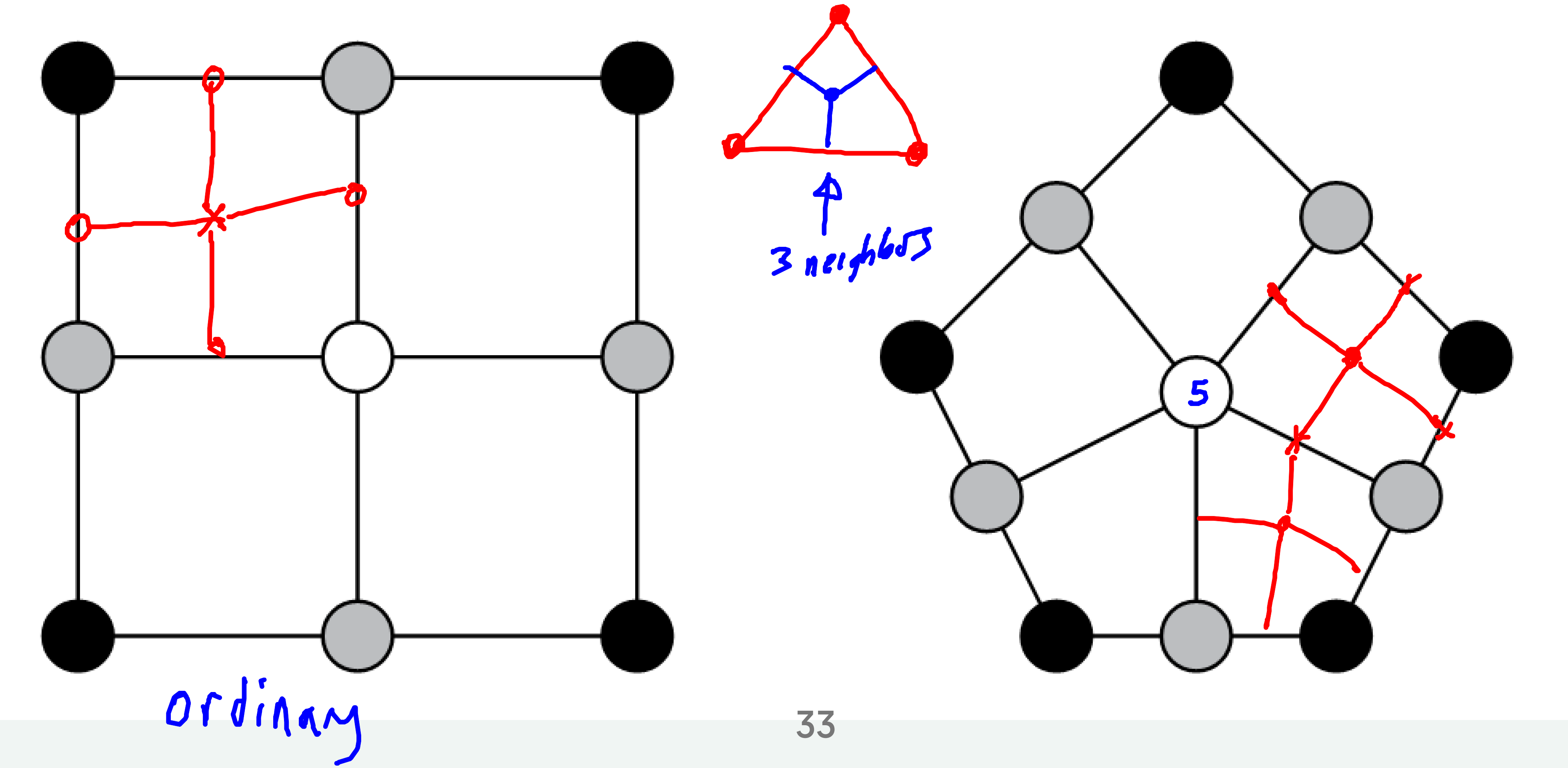
- Each iteration it gets smoother
- In the limit its a spline patch
- Can compute where each point will go

normals / tangents



Catmull-Clark

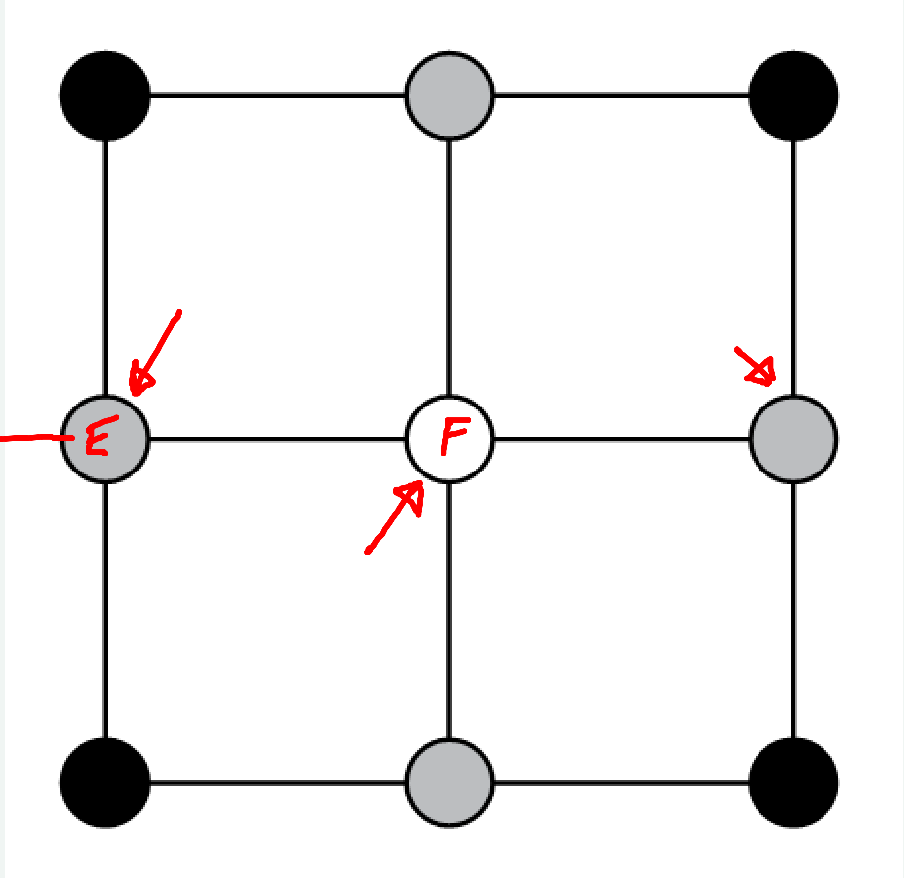
- Quads (everything is a quad after 1 iteration)

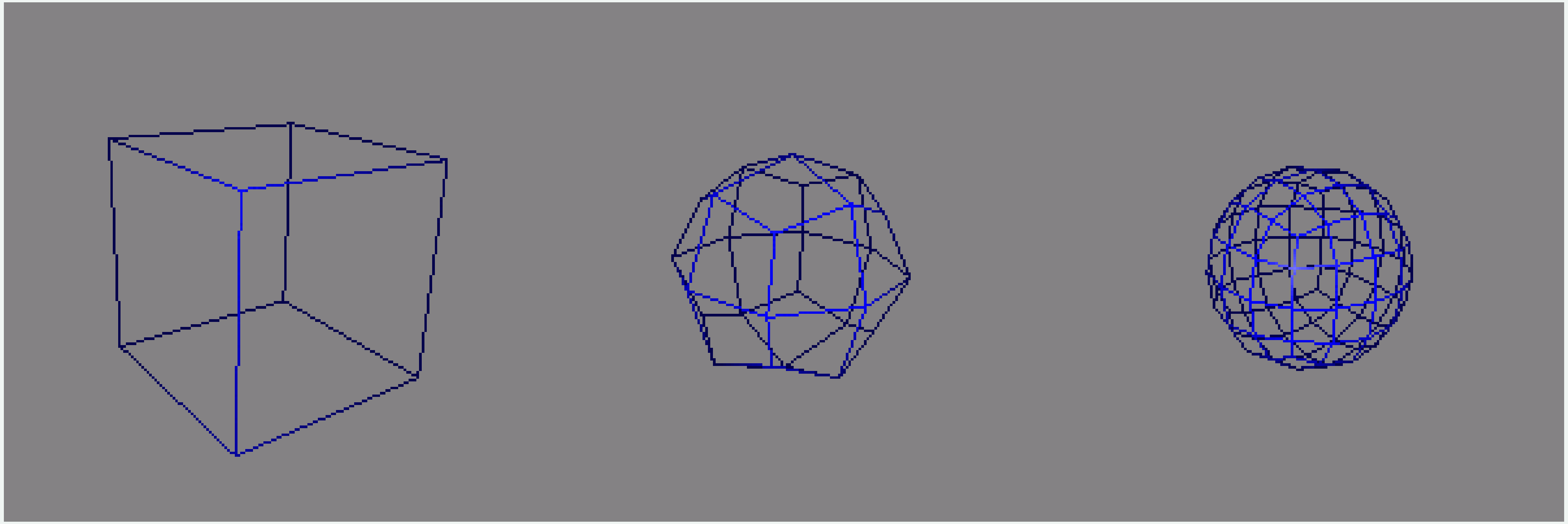


Catmull-Clark Rules

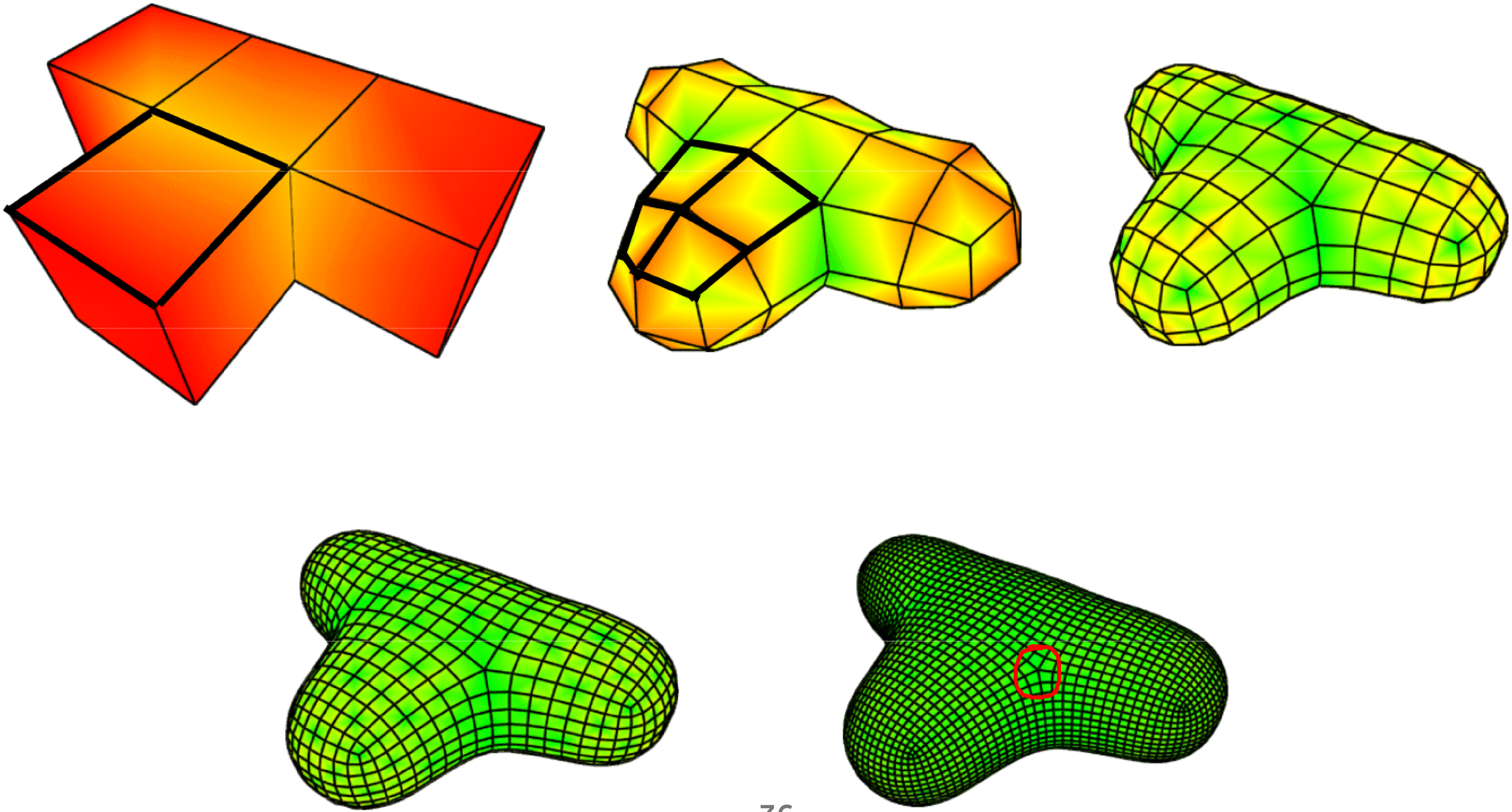
- Face Point = center of polygon
- Edge Point = average 4 neighbors (2 edge, 2 faces)
- Old Points (w/ N edges/faces)
  - $(n-2)/n$ times itself
  - $1/n^2$ average of N edges
  - $1/n^2$ average of N faces

F





http://www.holmes3d.net/graphics/subdivision/



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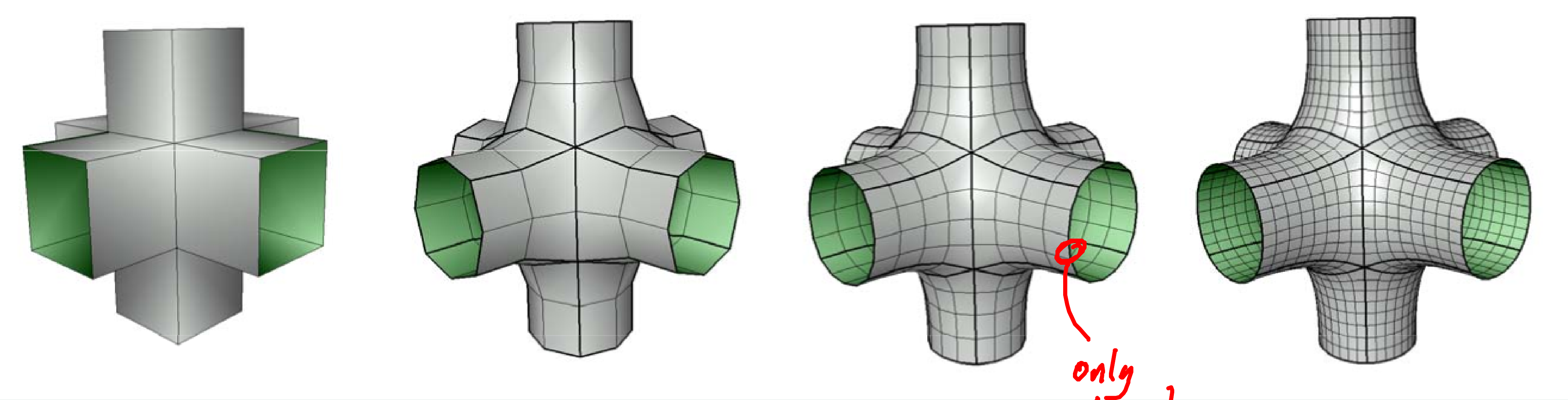
Quads Example



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only depend on edges

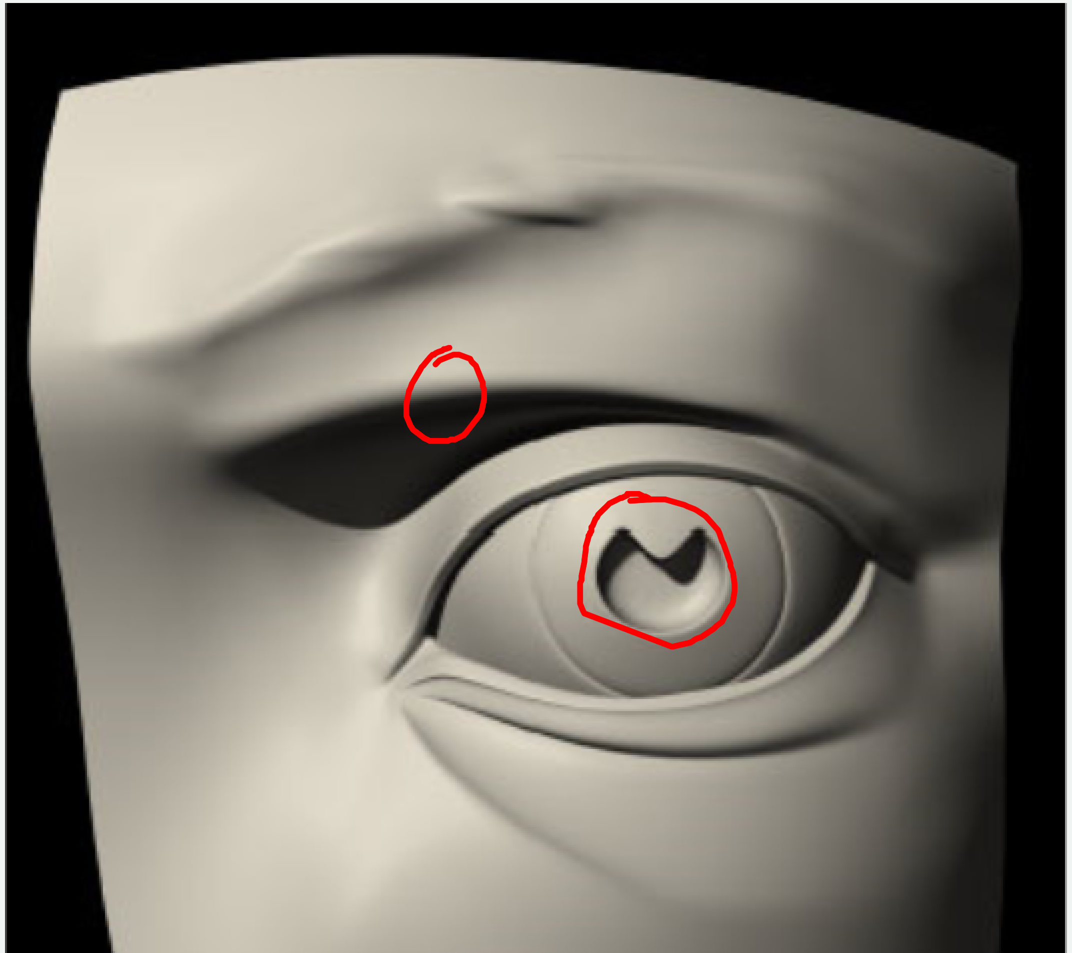
What About Edges?

Edges depend only on edges:

- causes them to be "regular curves"

Good Tricks (1) ...

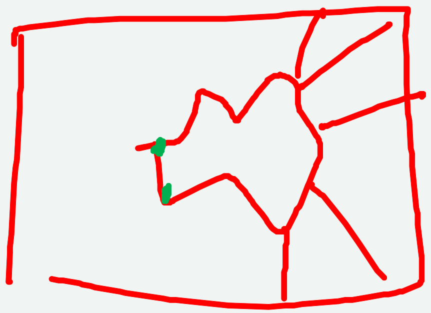
Creases - don't move points for some iterations

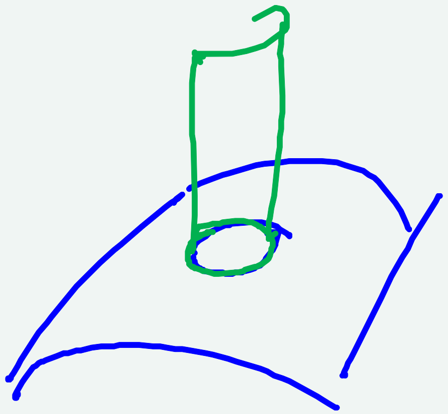


Good Tricks (2) ... Cutting and Sewing

Put a curve inside of a surface (hole or edge)

Curves stay curves - on any surface!



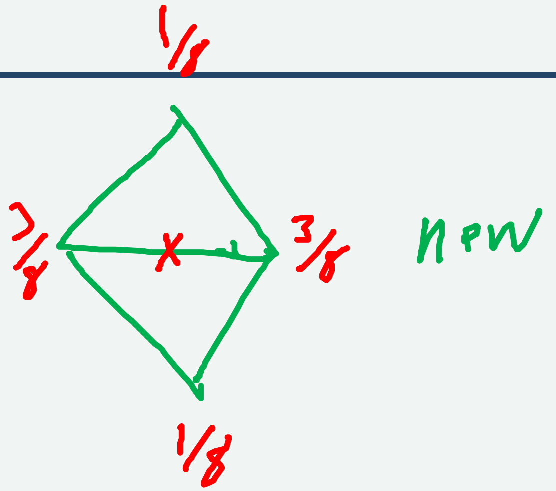


Why do we like Catmull-Clark so Much?

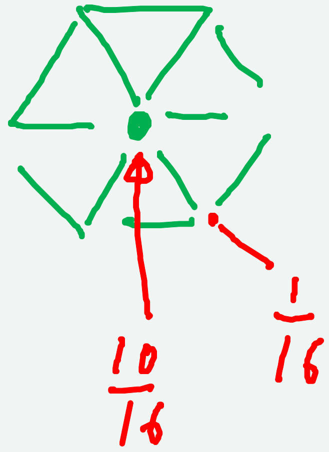
- Generalizes Cubic B-Splines
- Allows for stopping at any time
- Can compute exact normals (since B-Splines)
- Much easier than Non-Subdivision
- Not that hard to implement
  - requires mesh data structures for splitting and neighbor finding
- Made Popular by Pixar

(Smooth) Surfaces Review

Loop



- Surface vs. Solid Vs. Curve
- Not Free-Form
  - primitive shapes
  - generalized primitives (sweeps, lofts, ...)



- Free Form
  - Implicit
  - Parametric (and why not)
  - Subdivision (why and how)