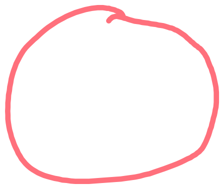
Lecture 27
Surfaces

Surface Modeling

Flat surfaces (or piecewise flat)

- polygons, triangles
- meshes



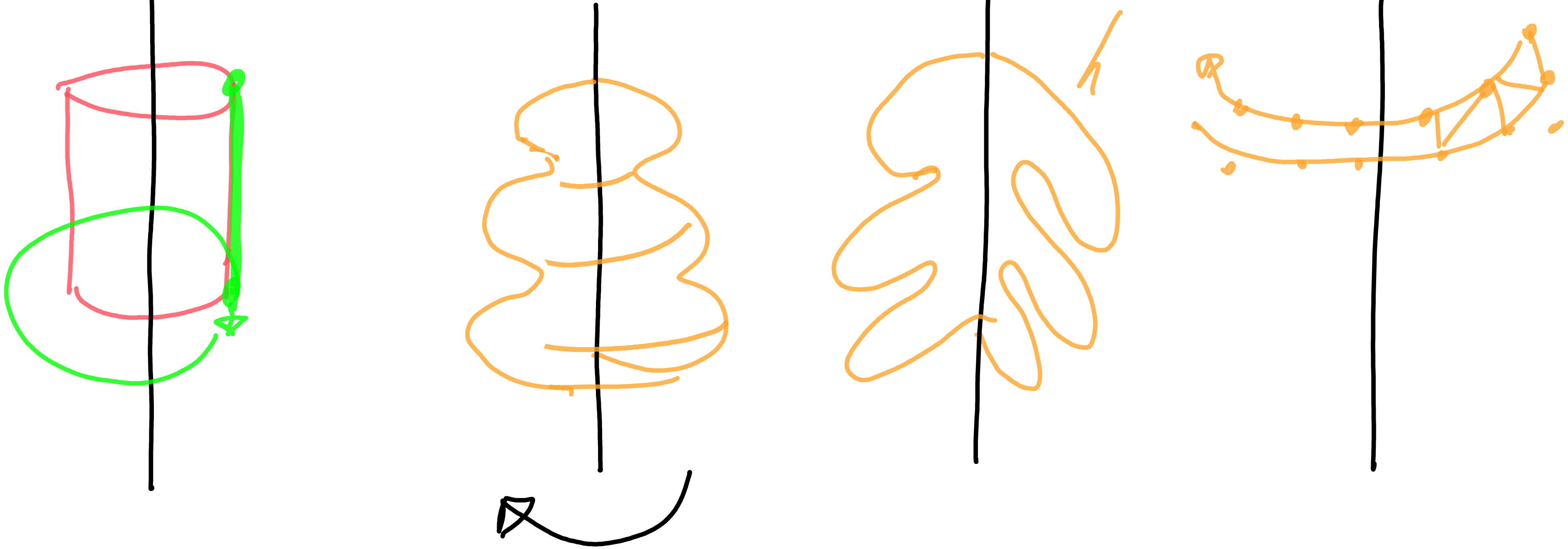
Standard shapes

- cone, cylinder, sphere (ball is volume)
- more complex (surfaces of revolution, generalized cylinders)
- and many more...

Free Form Surfaces

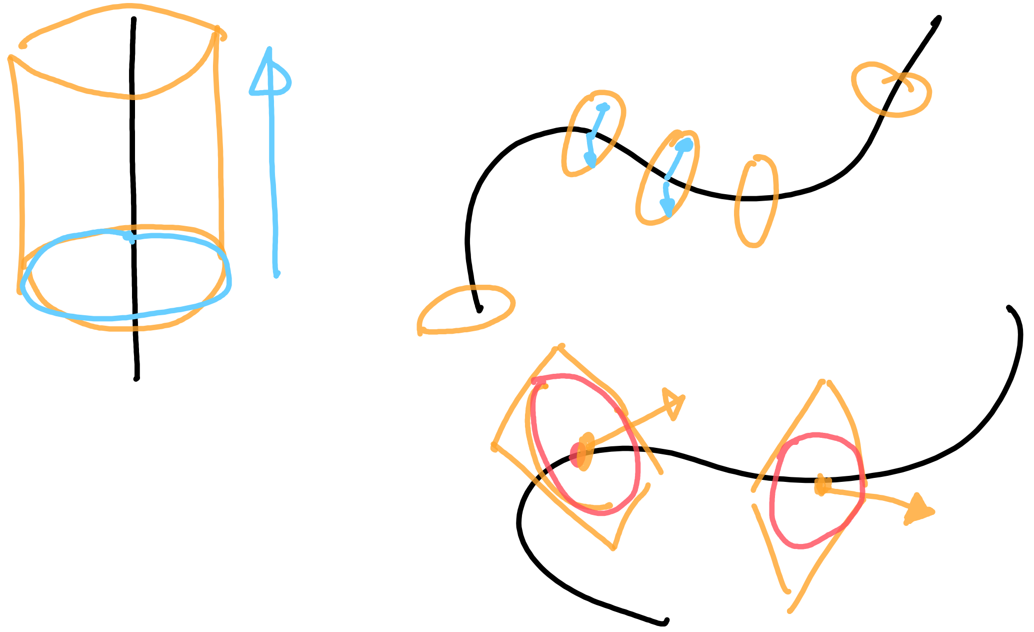
Surface of Revolution

1. Define a 2D Shape
2. Revolve it around an axis



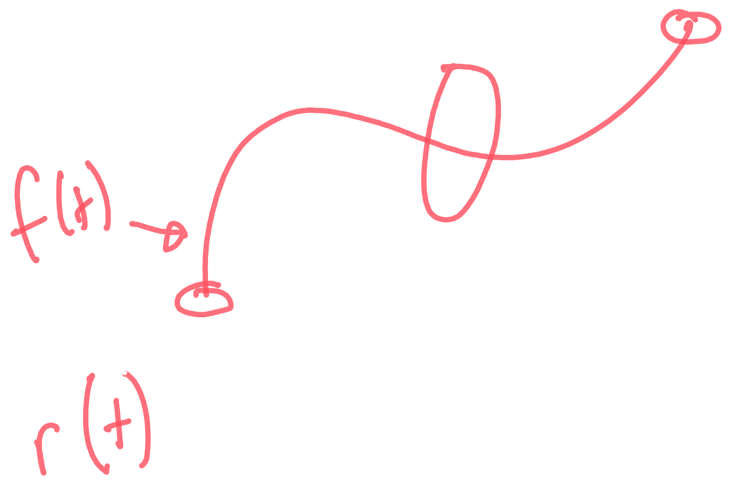
Generalized Cylinders (1) Tubes

1. Define a spine (function of t)
2. Give a radius



Generalized Cyliders (2) Cones

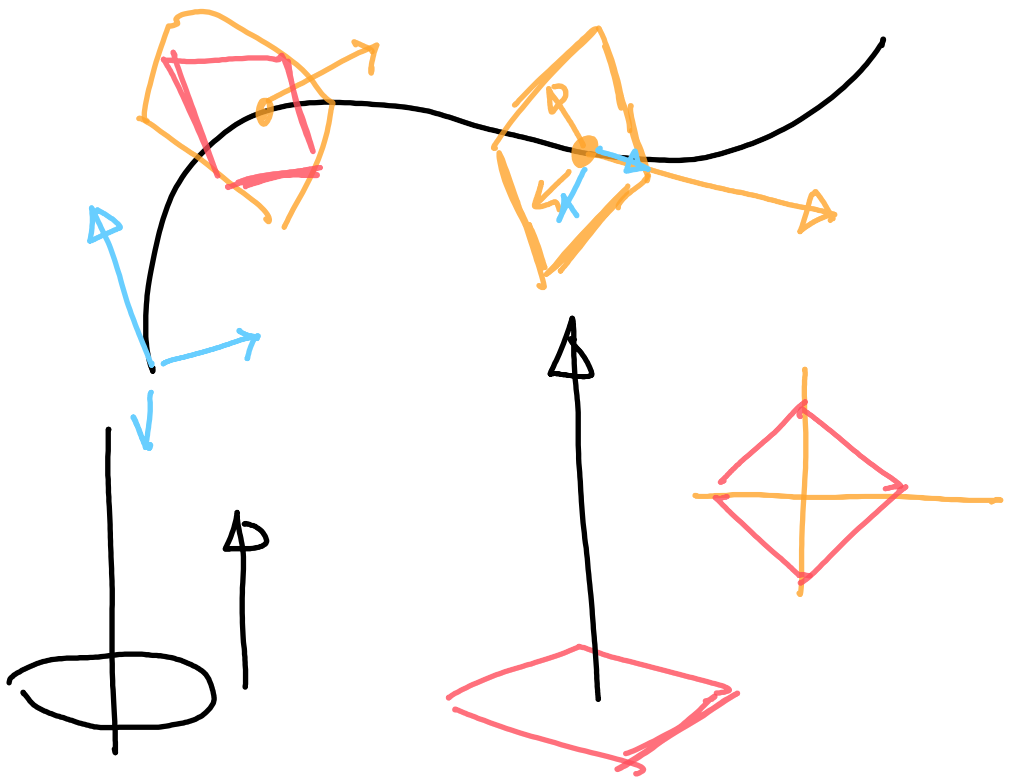
1. Define a spine (function of t)
2. Define a radius (function of t)



Generalized Cylinders (3) Sweeps

1. Define a spine
2. Define a cross-section shape

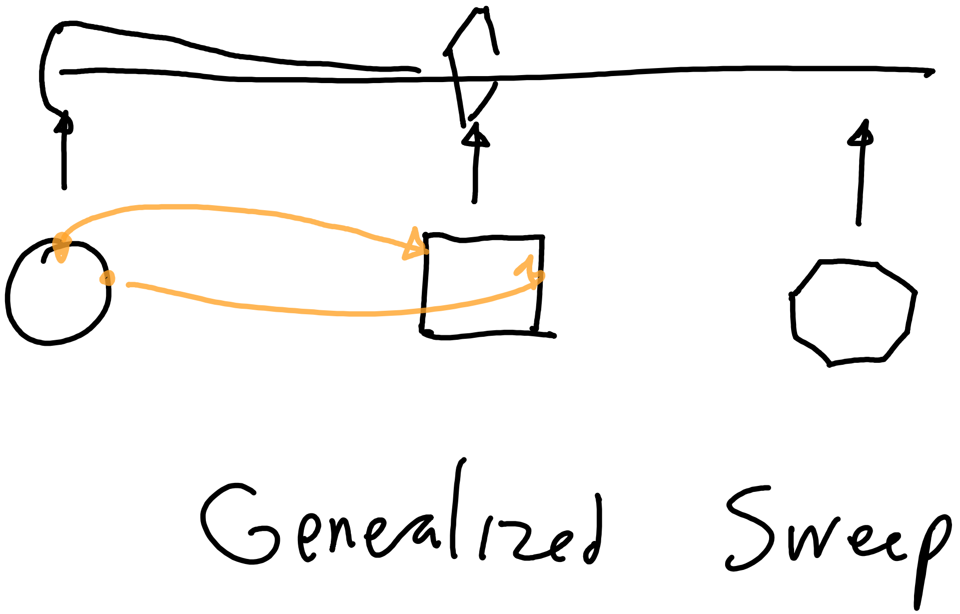
Frenet Frame
2nd deriv ←



Fancy Sweeps

2D Shape interpolation along spine

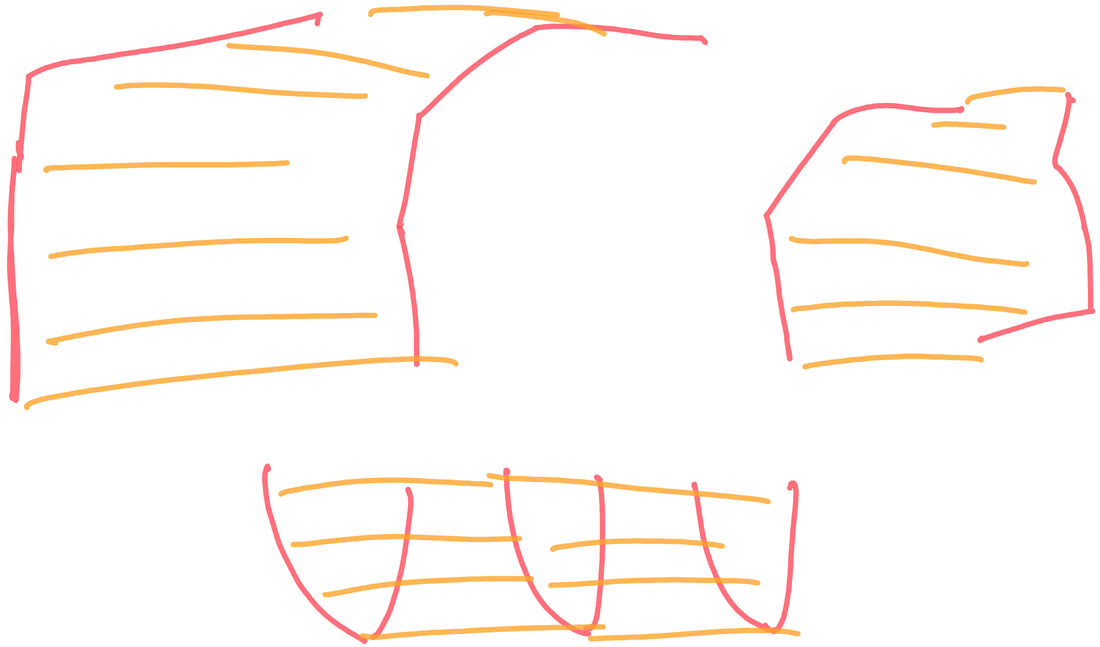
Requires good 3D curves



Lofting and Other Shape Methods

Define surfaces by curves

Interpolate between curves



Free Form Surfaces: Approaches

Same as curves

- Parametric: $(x, y, z) = \mathbf{f}(u, v)$ ← curves
- Implicit: $f(x, y, z) = 0$
- Procedural
- Subdivision ←

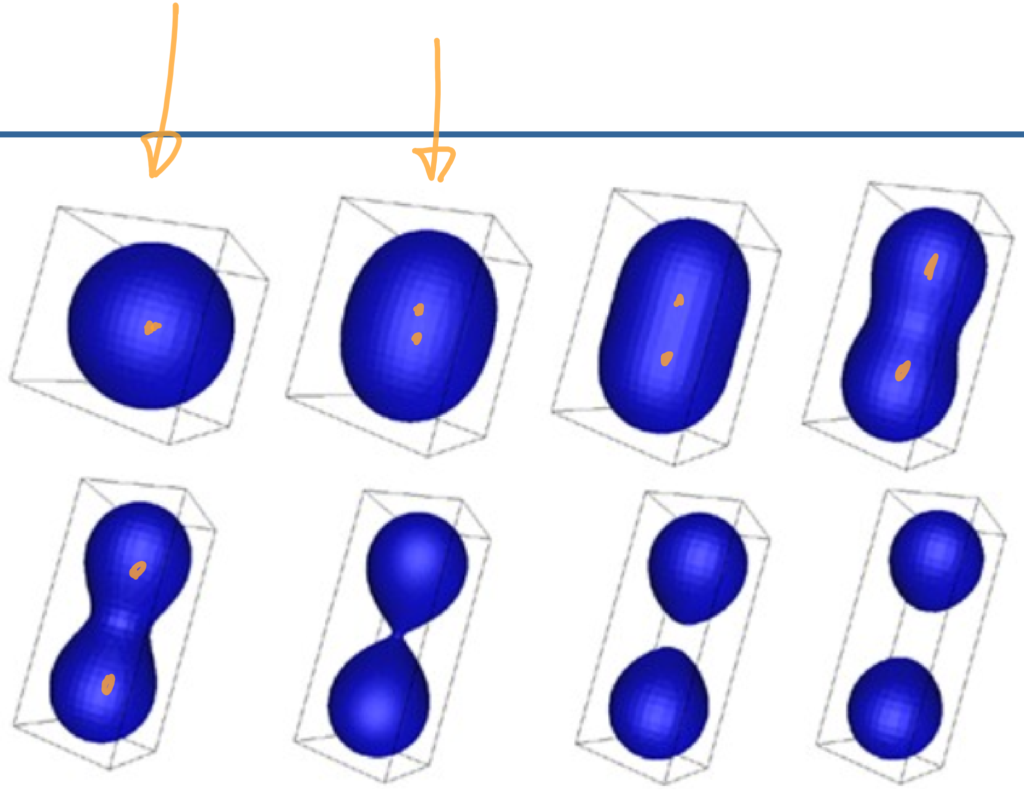
Implicit Surfaces

$f(x, y, z) = 0$

testing

Curve $f(x, y) = 0$

- sphere
- set of spheres
- distance to a set of points
- density (blobs)
  - (falls of to zero quickly)
- model by summing blobs



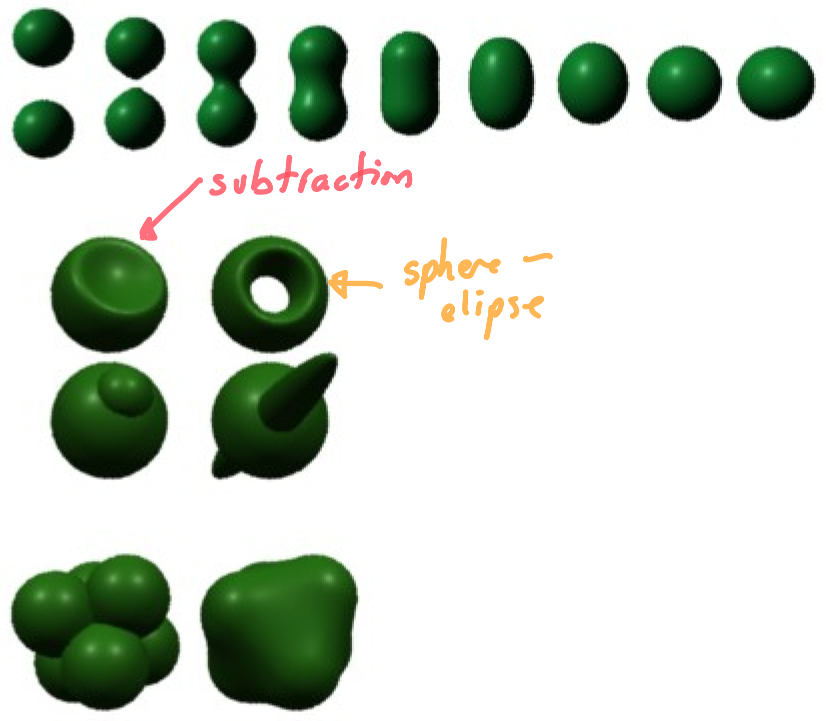
$x^2 + y^2 + z^2 + r^2 = 0$

How to draw an implicit surface?

Need to find points on $f(x, y, z) = 0$

Why do we like this?

Easy to combine simple units



Free form surfaces - Parametric

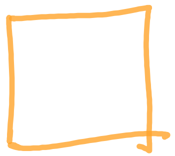
Is there an analog to polynomial curves?

$$f(u) \to \mathcal{R}^3$$

$u \in [0,1]$

Parametric Surfaces:

$$f(u, v) \to \mathcal{R}^3$$



Cubic Polynomials

curve: $f(u) = a_0 + a_1 u^1 + a_2 u^2 + a_3 u^3$

surface: $f(u, v) = ???$

Polynomial in u and v! (tensor product)

$$\begin{aligned} f(u,v) = & a_{00}u^0v^0 + a_{01}u^1v^0 + a_{02}u^2v^0 + a_{03}u^3v^0 + \\ & a_{10}u^0v^1 + a_{11}u^1v^1 + a_{12}u^2v^1 + a_{13}u^3v^1 + \\ & a_{20}u^0v^2 + a_{21}u^1v^2 + a_{22}u^2v^2 + a_{33}u^3v^2 + \\ & a_{30}u^0v^3 + a_{31}u^1v^3 + a_{22}u^2v^3 + a_{33}u^3v^3 \end{aligned}$$

Tensor Product Surface Patches

16 coefficients (control points)!

$$
\begin{aligned}
f(u,v) = & a_{00}u^0v^0 + a_{01}u^1v^0 + a_{02}u^2v^0 + a_{03}u^3v^0 + \\
& a_{10}u^0v^1 + a_{11}u^1v^1 + a_{12}u^2v^1 + a_{13}u^3v^1 + \\
& a_{20}u^0v^2 + a_{21}u^1v^2 + a_{22}u^2v^2 + a_{33}u^3v^2 + \\
& a_{30}u^0v^3 + a_{31}u^1v^3 + a_{22}u^2v^3 + a_{33}u^3v^3
\end{aligned}
$$

There are analogs to curve formulations

- Bezier, B-Spline, Interpolating, ...

patche patchs patchs,

Tensor Product Surfaces are Hard!

How to connect two patches?

- Continuity
- Stitching together

How to cut a patch?

- Make a Hole?
- Make an edge? (attachment)

How about non-square domains?

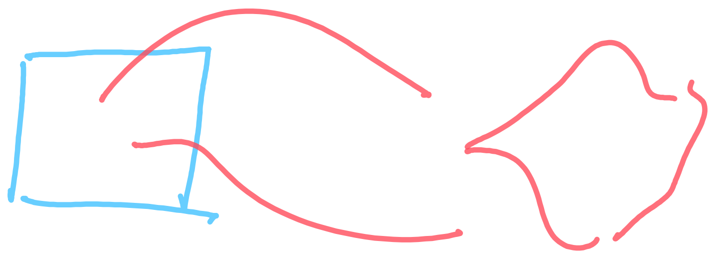
- inconvenient stretching?
- different topology?

What do we do instead?

Subdivision Surfaces!

Subdivision: Motivation

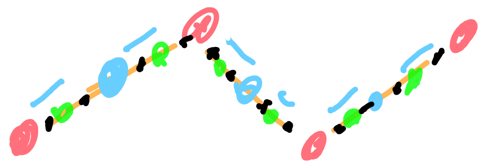
Polynomial Surfaces Are Challenging



f(u,v) -> x,y,z

- What if the patches aren't square?
- How do we connect them? (for smoothness)
- How do we cut holes in them?
- How do we stitch them together?

Subdivision: Intuitions from 2D



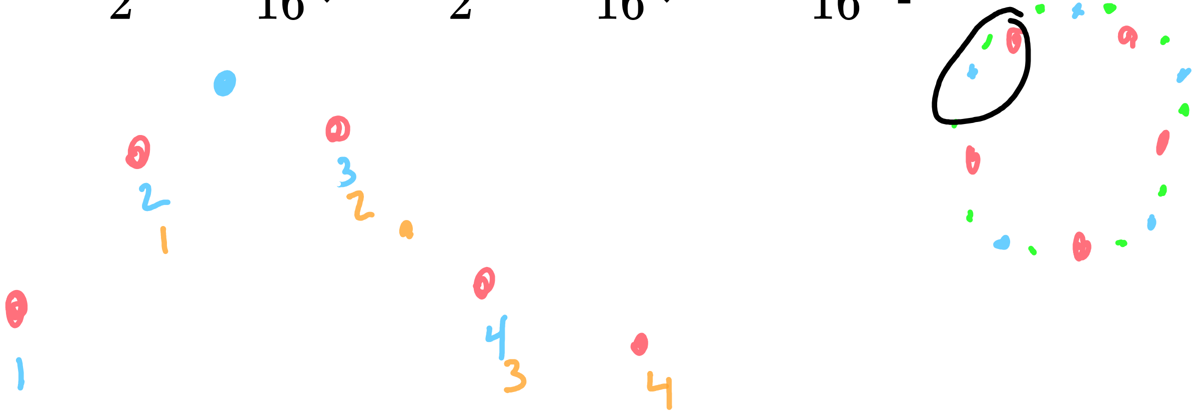
- Start with a set of (points) line segments
- Add new points / move old points
- Divide segments into more segments
- Repeat
  - until good enough
  - infinitely many times

Design so it converges to a smooth curve

Example 1: Dyn/Levin/Gregory

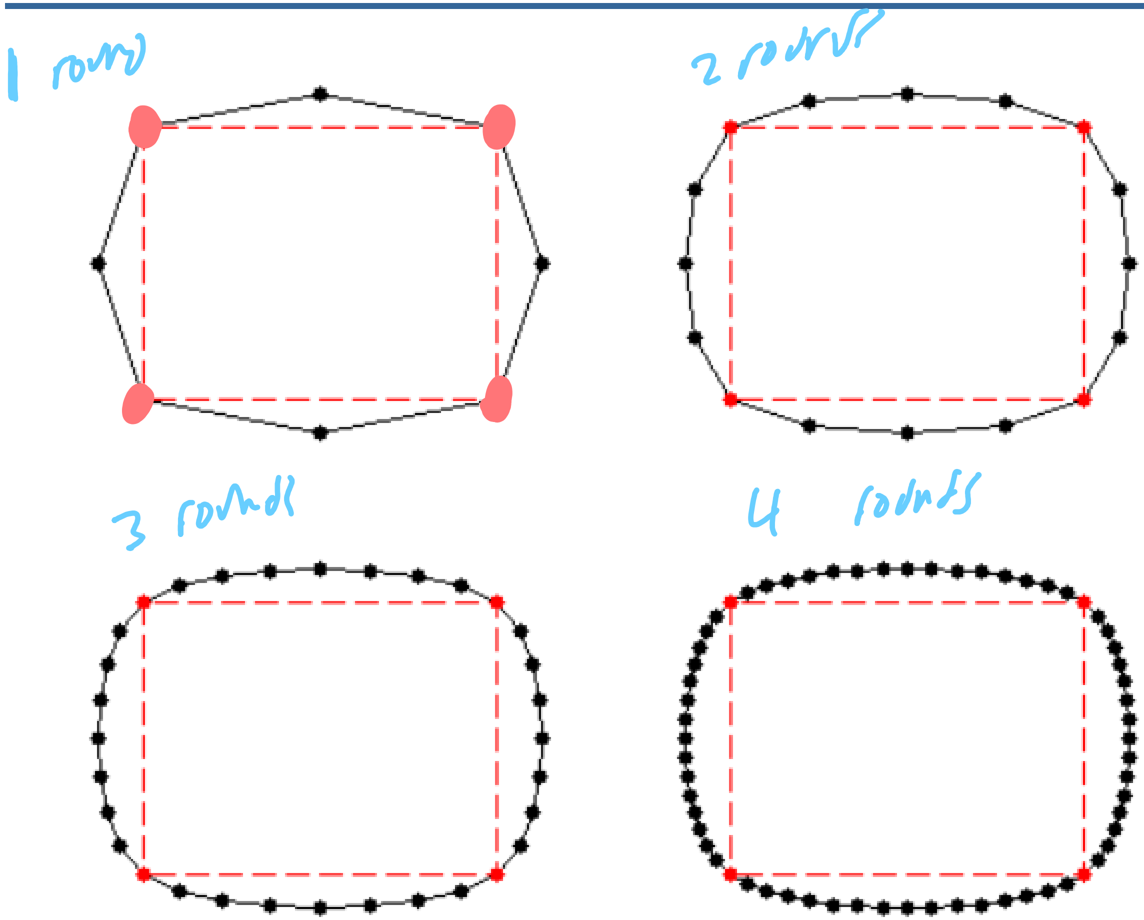
4 point scheme - each new point looks at 4 neighbors

[$-\frac{1}{16}$, $\frac{1}{2}+\frac{1}{16}$, $\frac{1}{2}+\frac{1}{16}$, $-\frac{1}{16}$]



more generally [$-w$, $\frac{1}{2}+w$, $\frac{1}{2}+w$, - $\frac{1}{16}$ $-w$]

Each time it gets smoother…



Infinitely many times?

Converges to a cubic spline! LIMIT CURVE

(you can read the proof)

Note: Interpolation

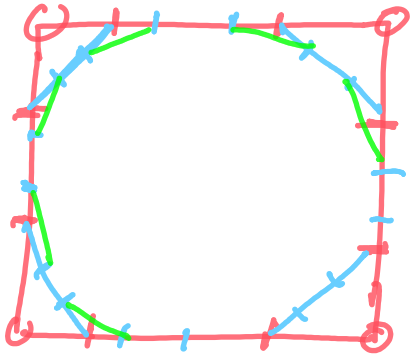
Original points continue - forever

Example 2: Not interpolating

Chakin Corner Cutting

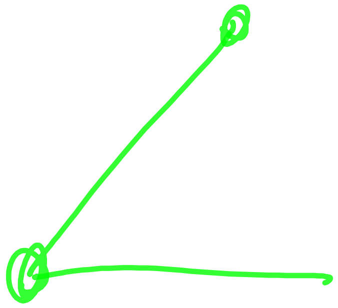
- each corner -> 2 points (1/4 from edge)
- each segment cut at (1/4, 3/4)

Converges to quadratic B-Spline



In 3D

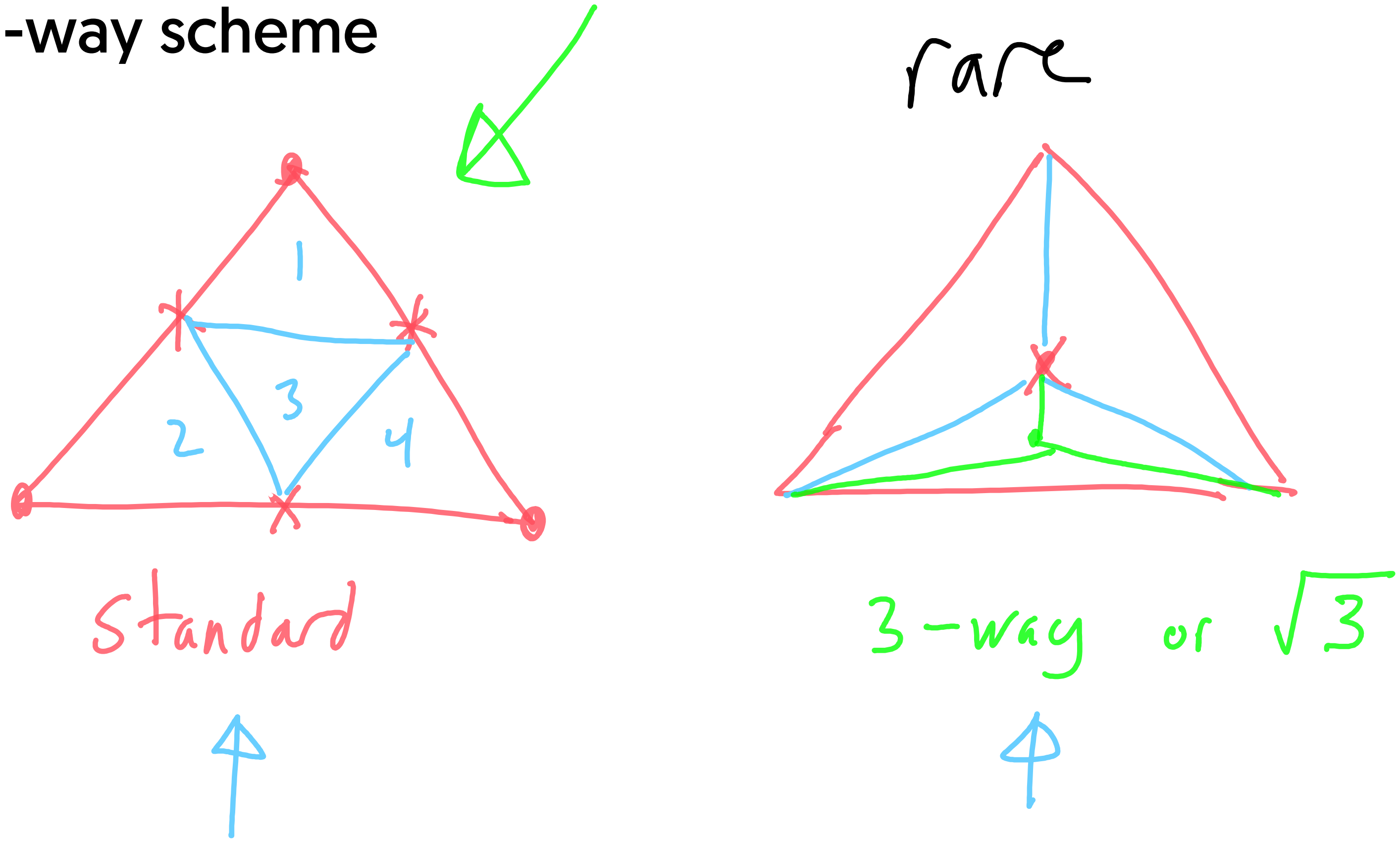
- Cut each triangle into new triangles
  - place the new vertices
  - move the old vertices (non-interpolating)



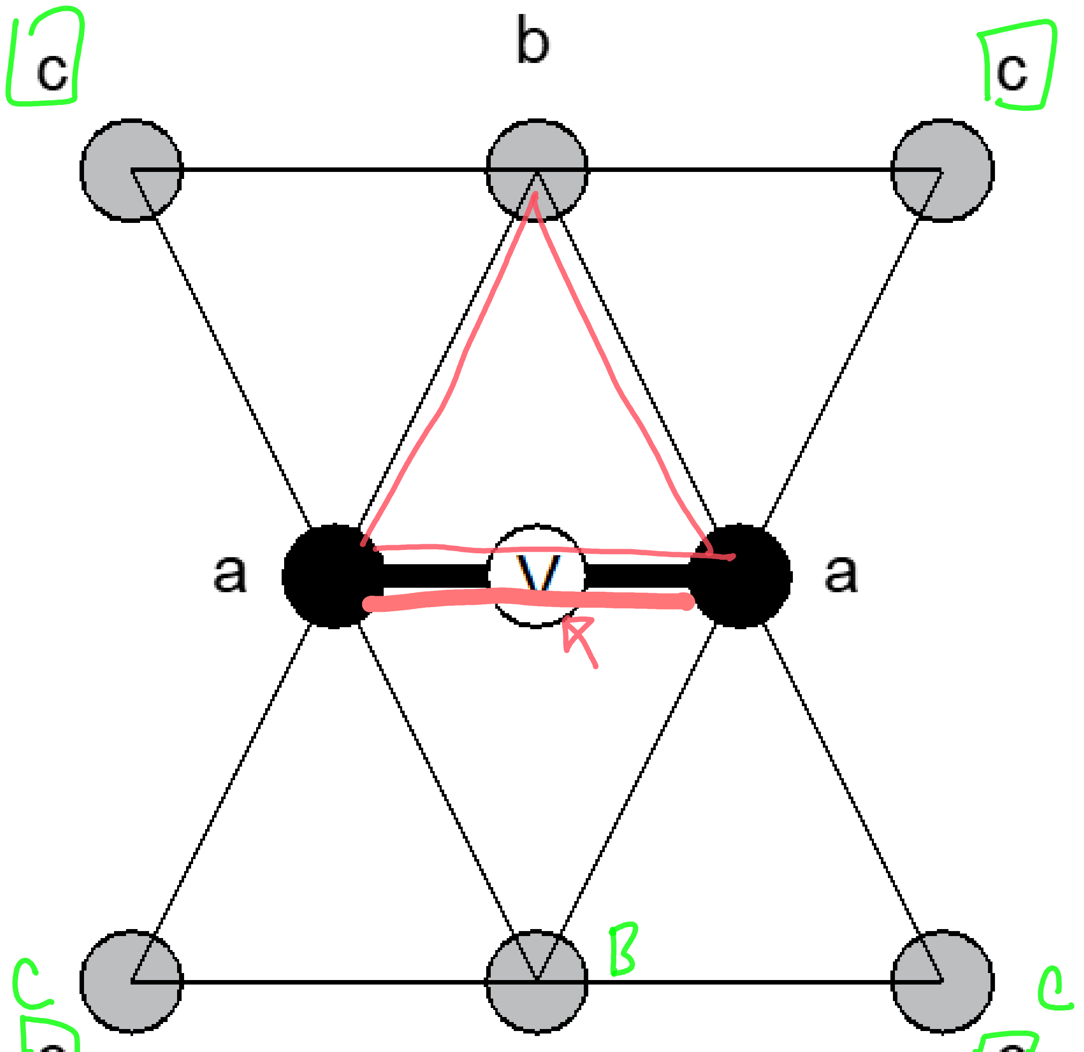
Dividing triangles

Standard (4-way) scheme

3-way scheme



Butterfly



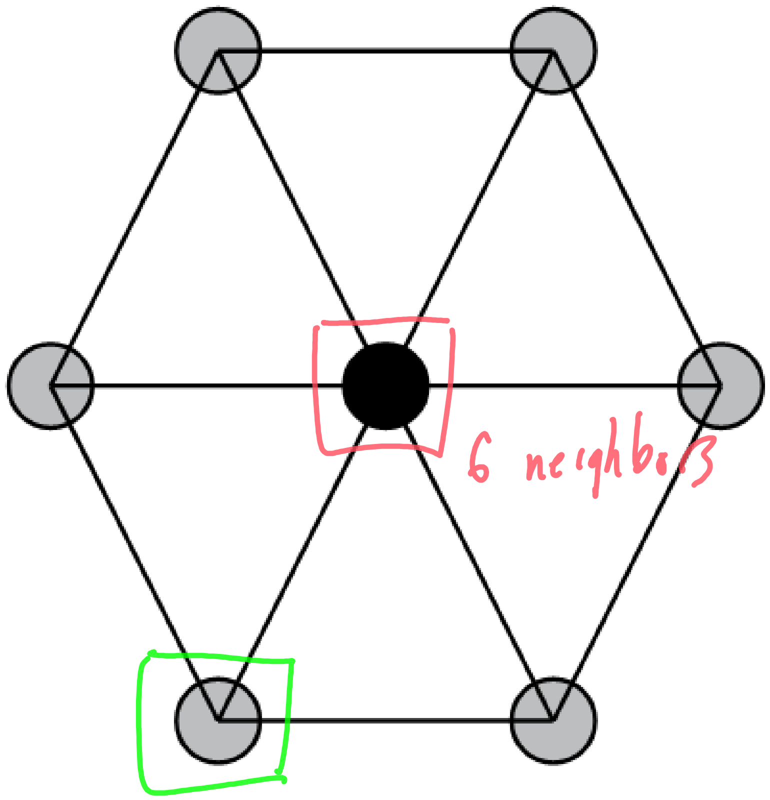
$$v = 1/2\,a + 1/8\,b - 1/16\,c$$

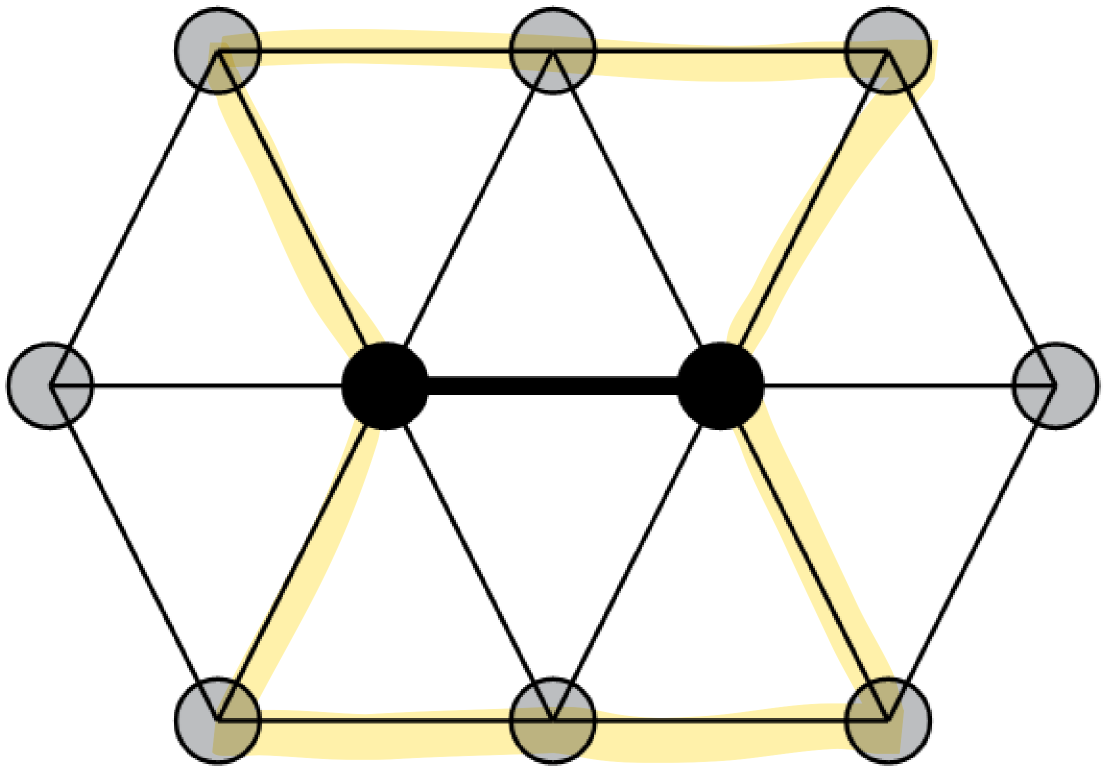
2 2 4

Uniform Meshes

Ordinary and Extra-Ordinary Points

not 6 neighbors

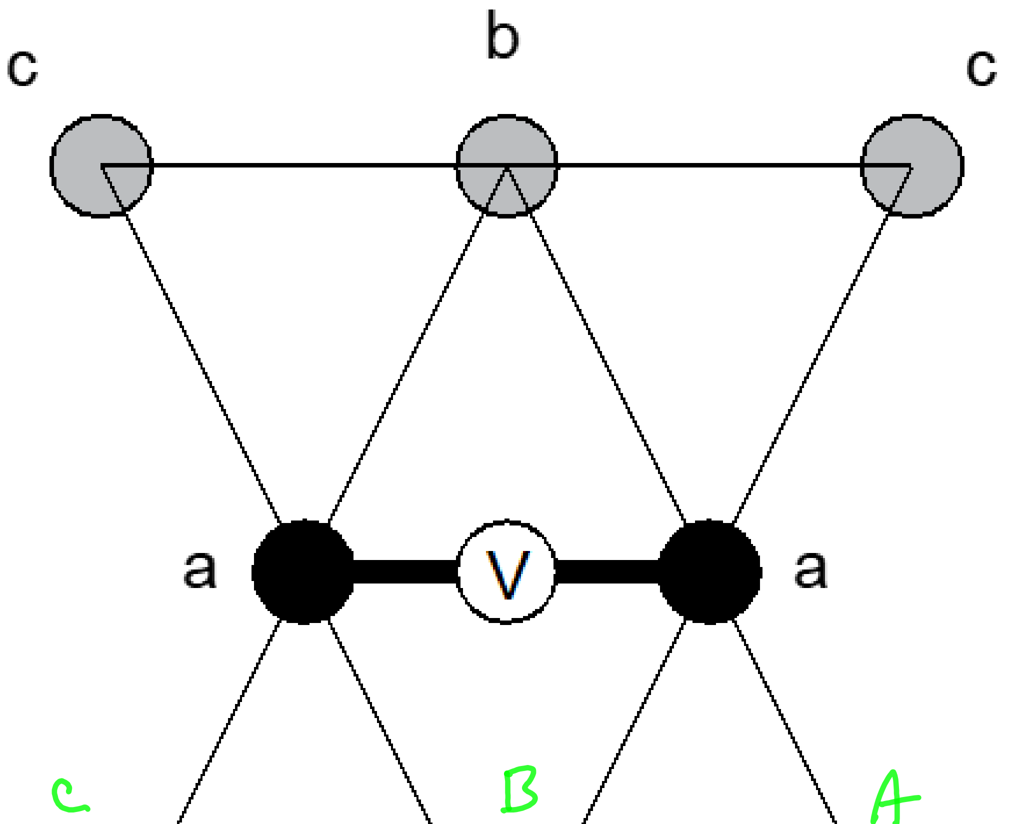




Butterfly

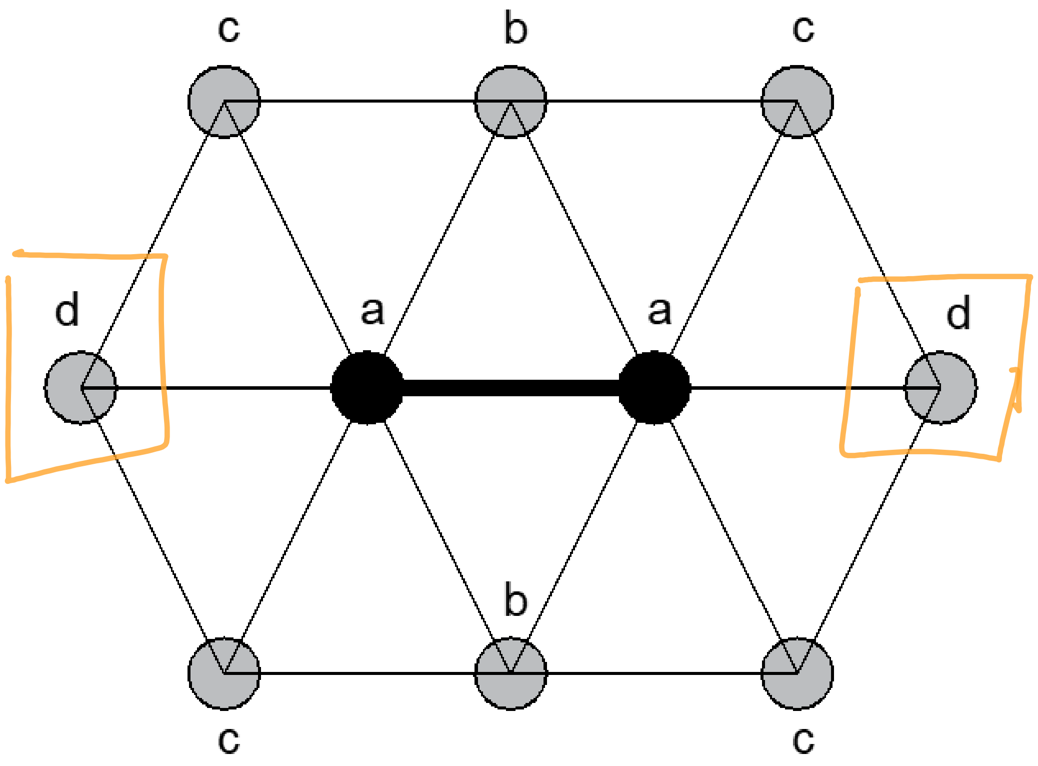
C[1] almost everywhere

Special rules for extra-ordinary points



$$v = 1/2\,a + 1/8\,b - 1/16\,c$$

Modified Butterfly

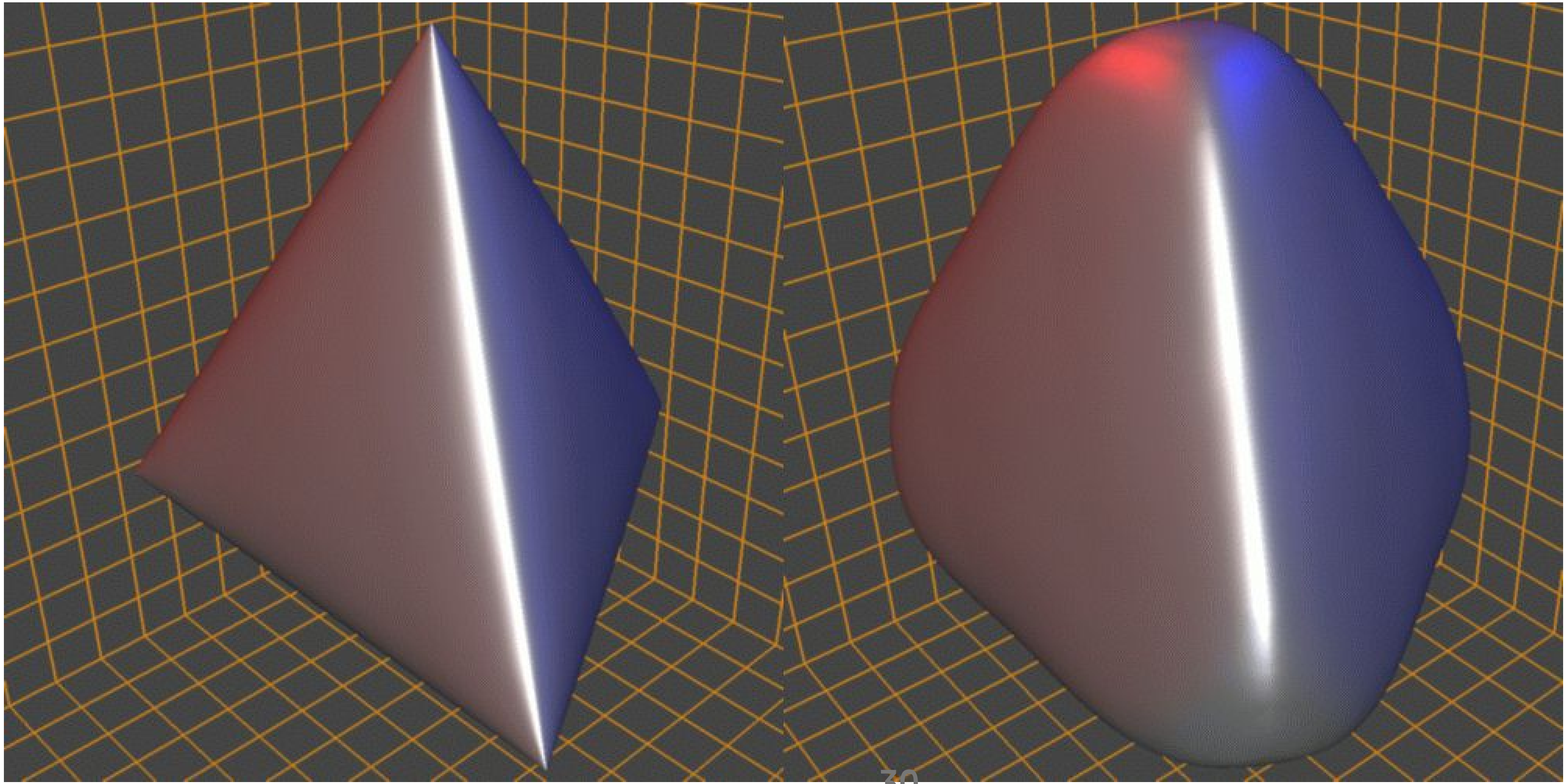


$\mathbf{v} = (1/2 - w)\,\mathbf{a} + (1/8 + 2w)\,\mathbf{b} - (1/16 - w)\,\mathbf{c} + w\,\mathbf{d}$

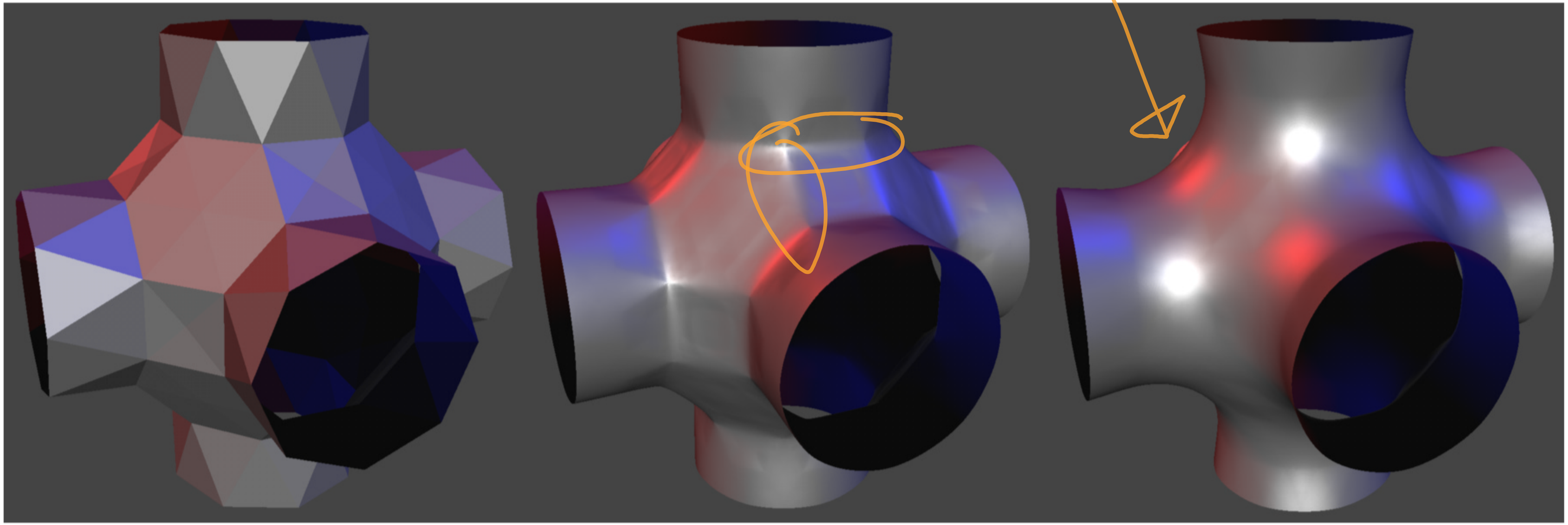
tension parameter w
sum over all 10 neighbors

$w = 0$
regular "butterfly"

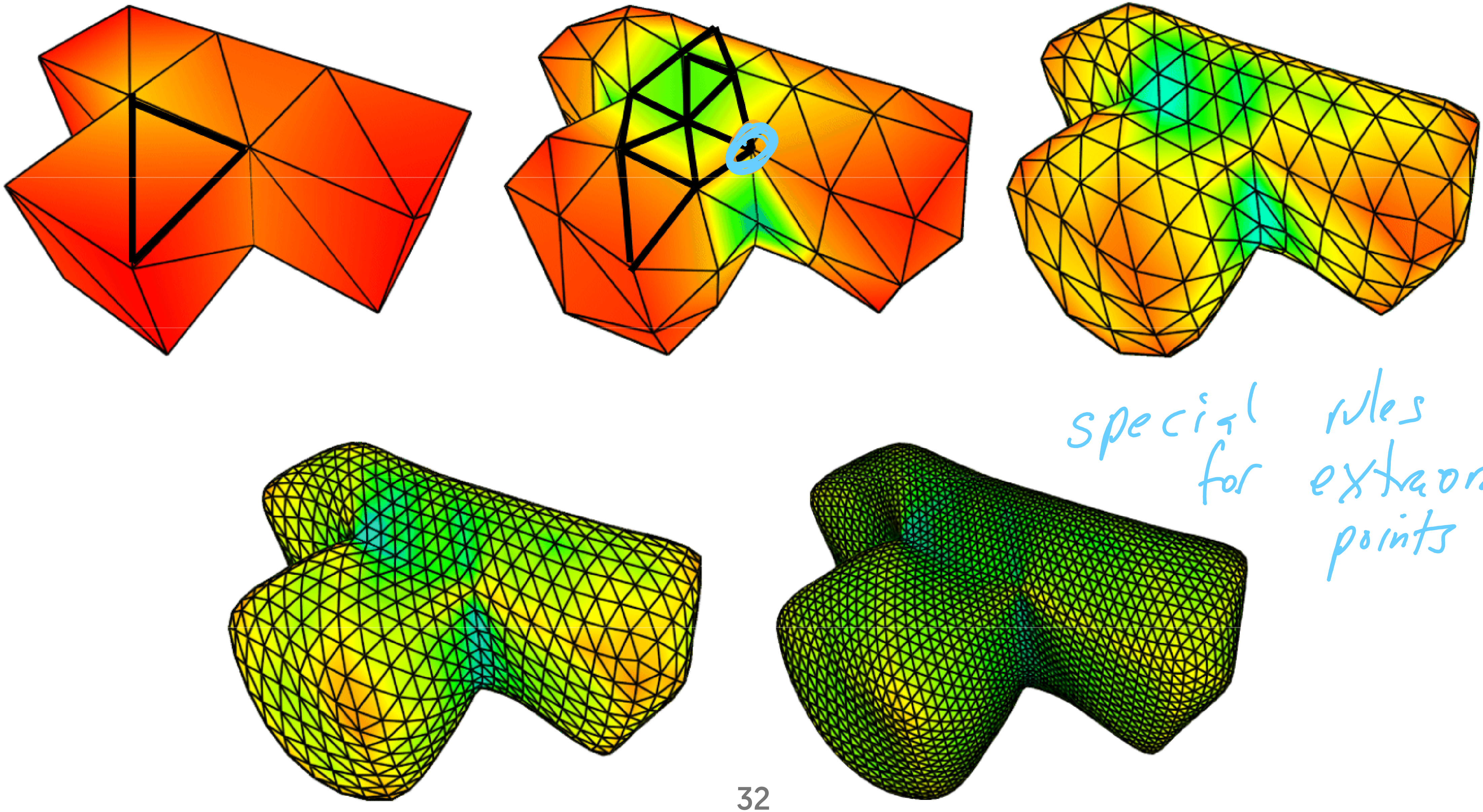
Tension



Butterfly vs. Modified



Initial mesh | Butterfly scheme interpolation | Modified Butterfly interpolation

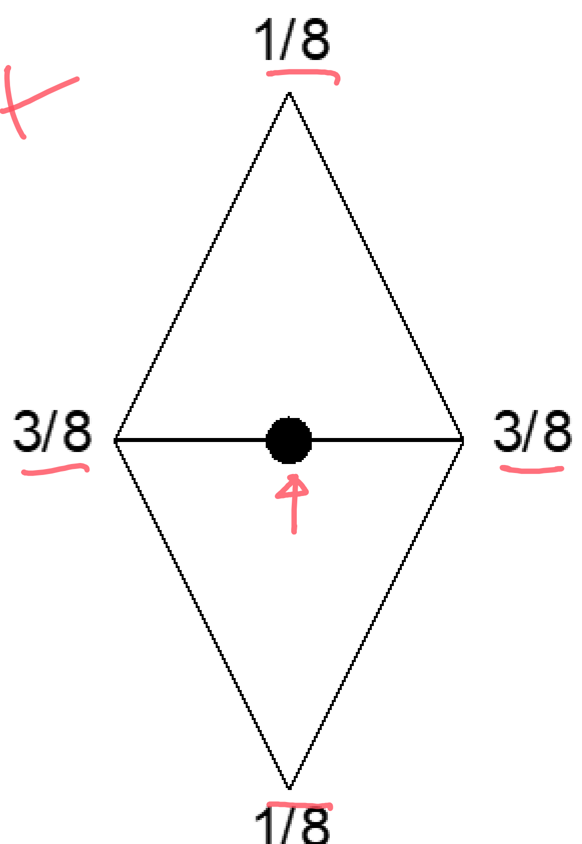


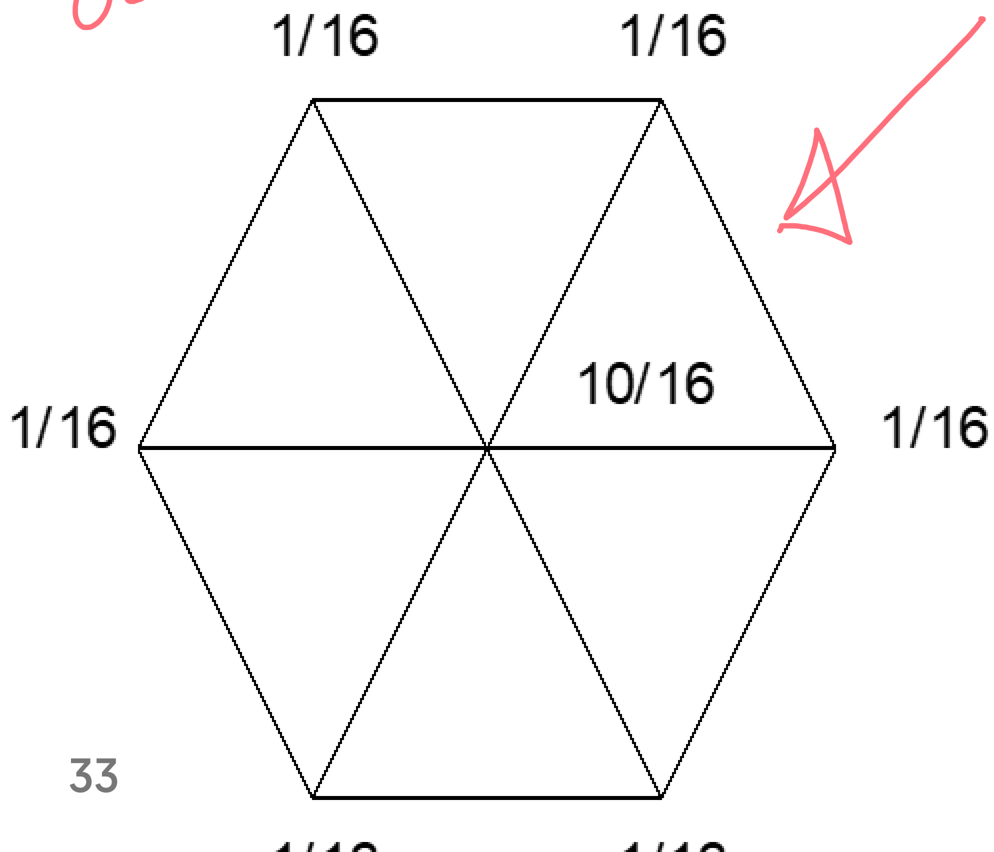
special rules for extraordinary points

http://graphics.stanford.edu/courses/cs468-10-

Loop Scheme

- New points split edges
- Old points moved to smooth

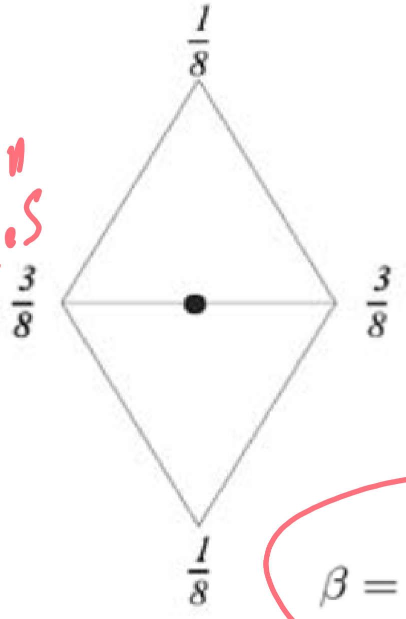




Loop Rules - General (irregular)

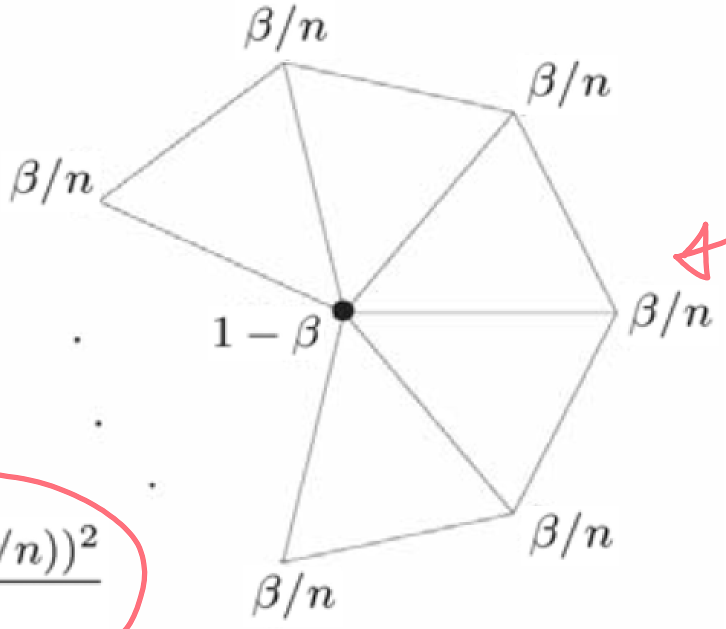
Full Loop rules (triangle mesh)

all edges between triangles



Interior

$$\beta = \frac{5}{8} - \frac{(3 + 2\cos(2\pi/n))^2}{64}$$

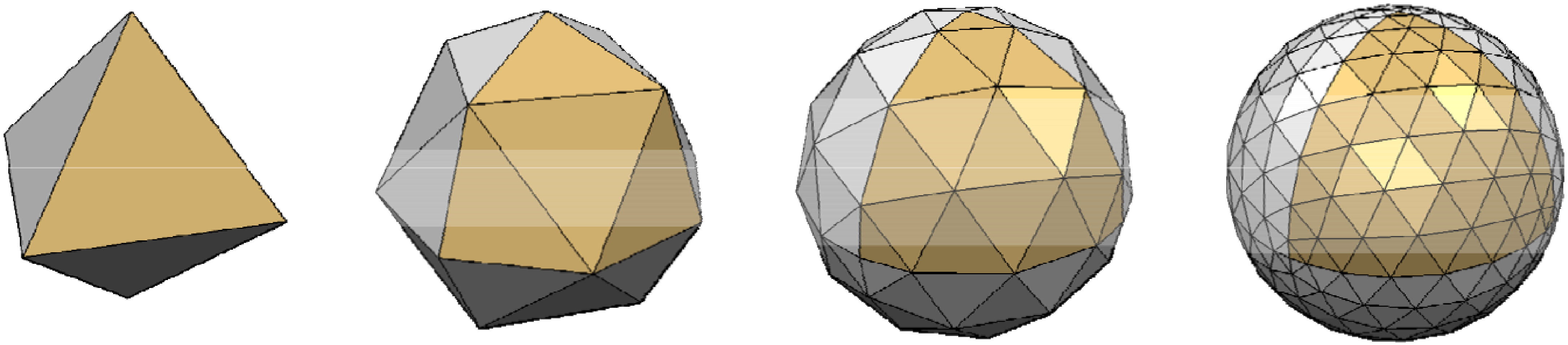


old point update

Loop Rules - Boundaries

- new points half way
- old points 1/8 3/4 1/8
- edges only depend on edges

Loop Example



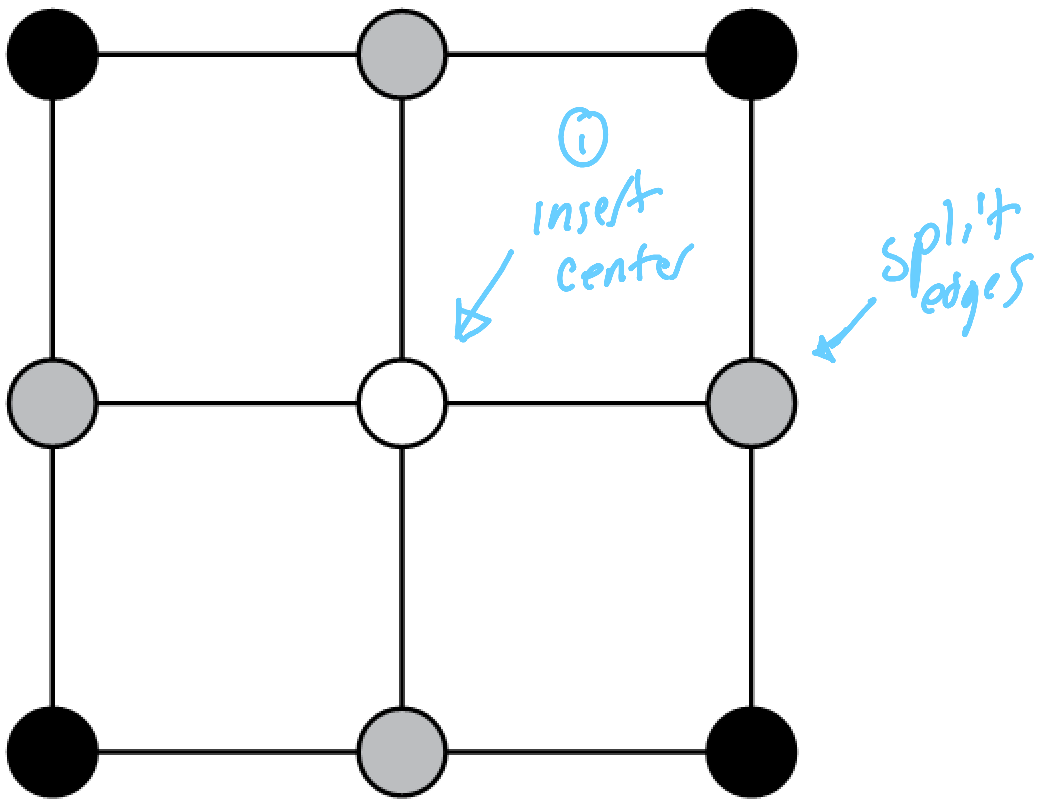
http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/10_Subdivision.pdf

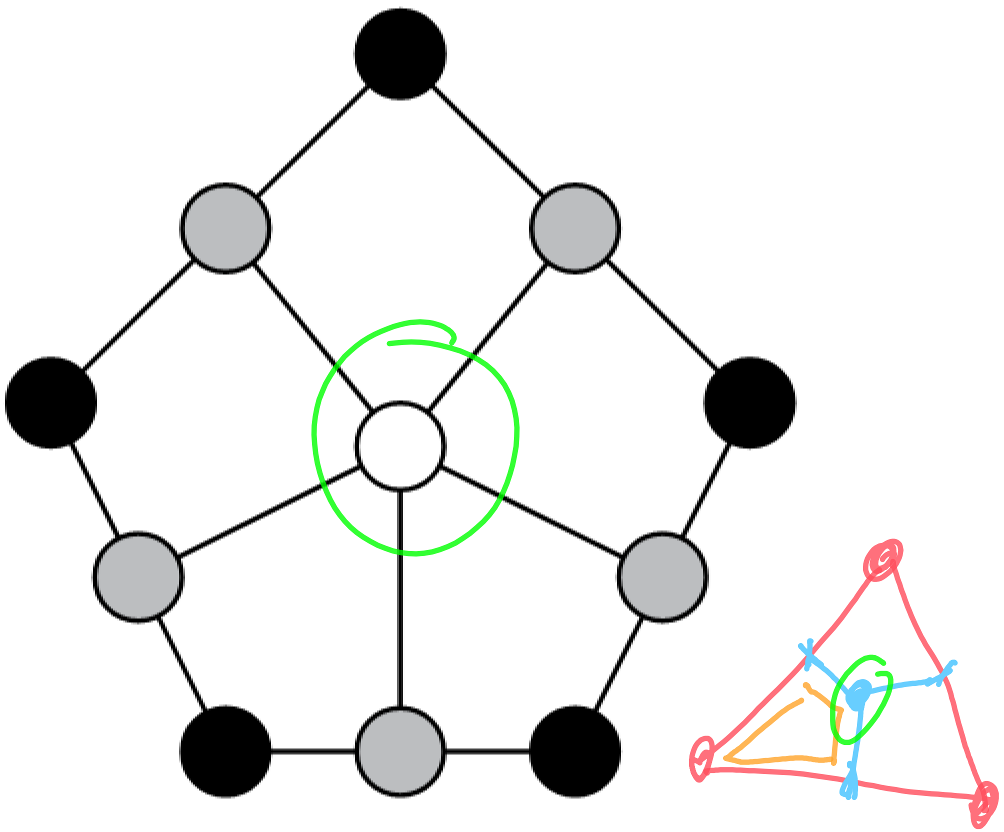
In the limit?

- Each iteration it gets smoother
- In the limit its a spline patch
- Can compute where each point will go

Catmull-Clark

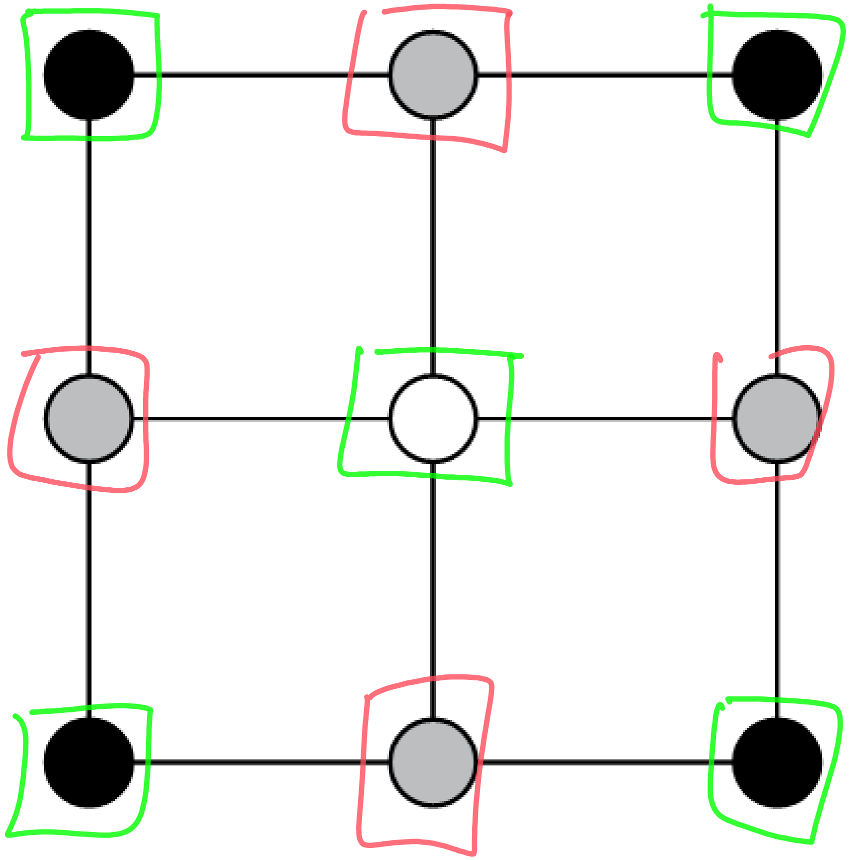
- Quads (everything is a quad after 1 iteration)

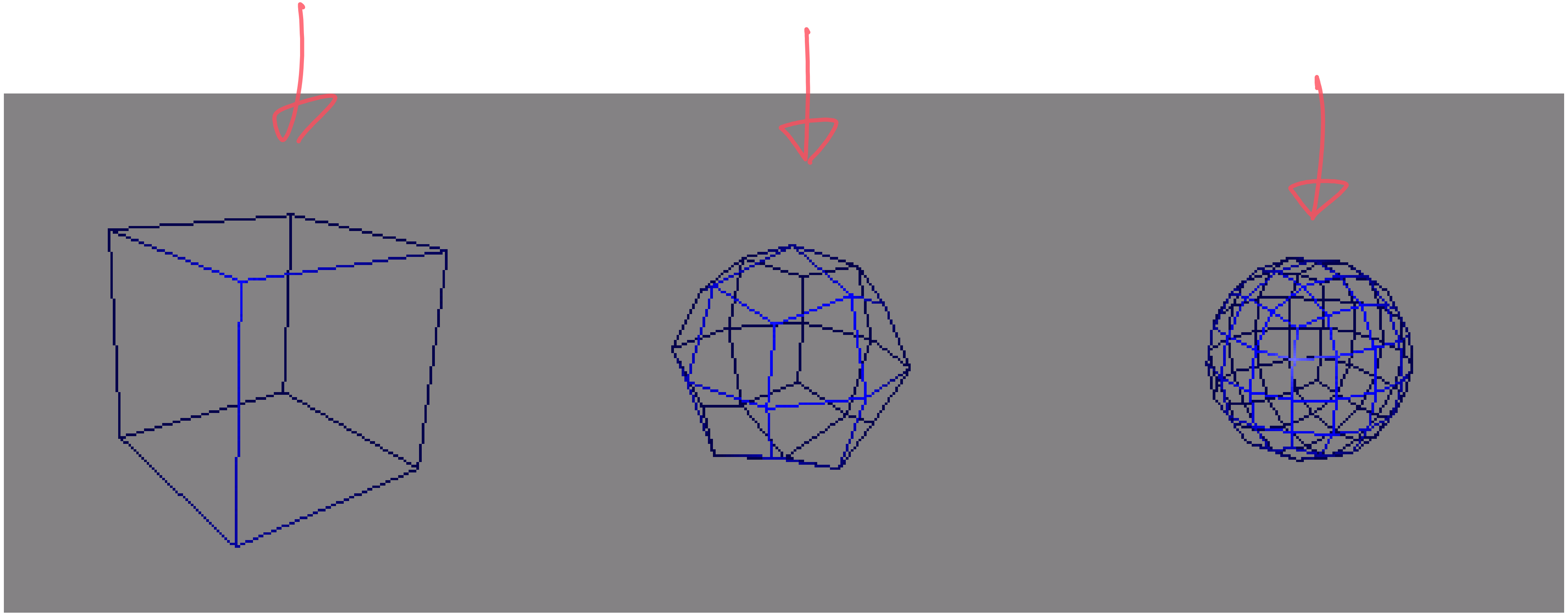




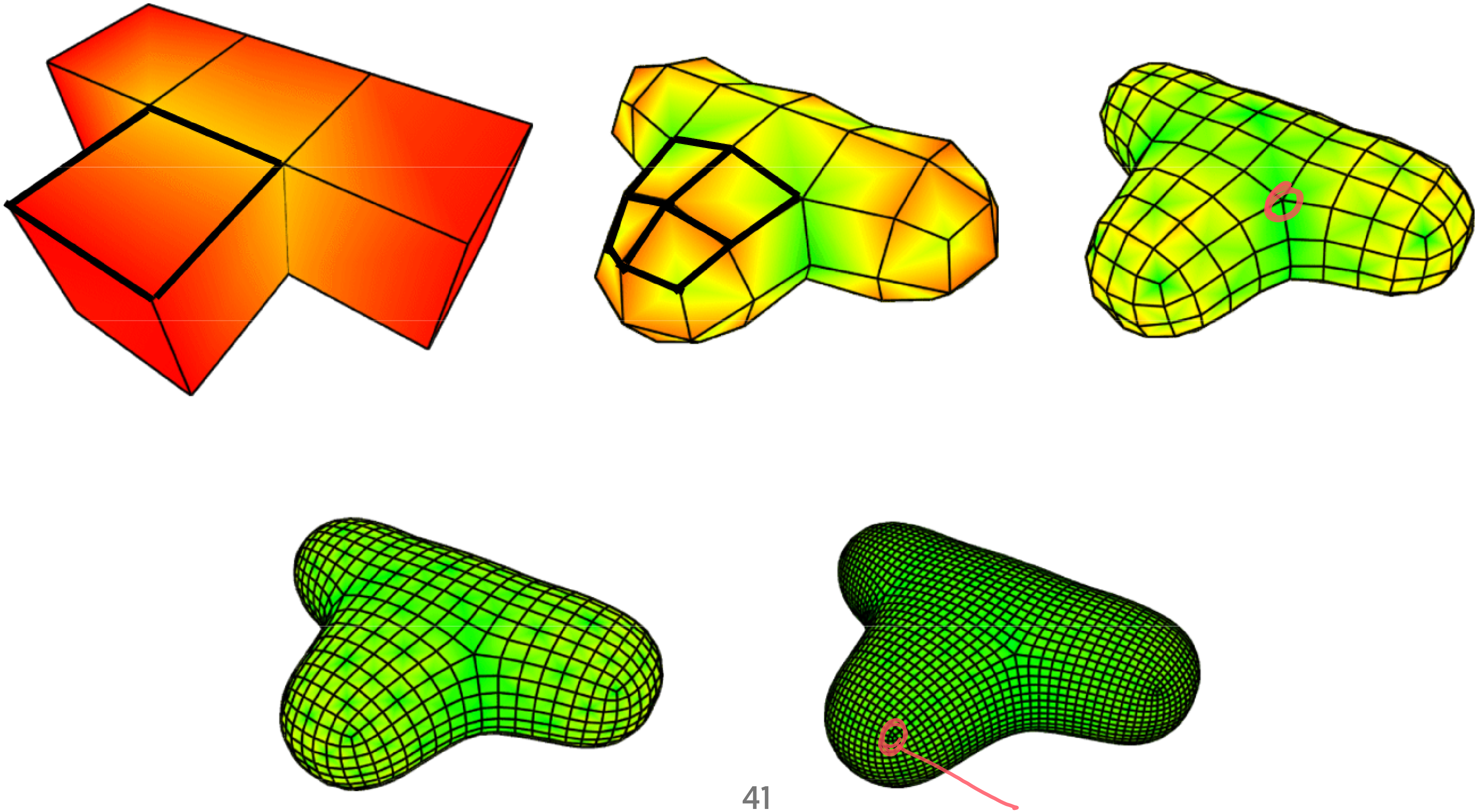
Catmull-Clark Rules

- Face Point = center of polygon
- Edge Point = average 4 neighbors (2 edge, 2 faces)
- Old Points (w/ N edges/faces)
  - $(n-2)/n$ times itself
  - $1/n^2$ average of N edges
  - $1/n^2$ average of N faces



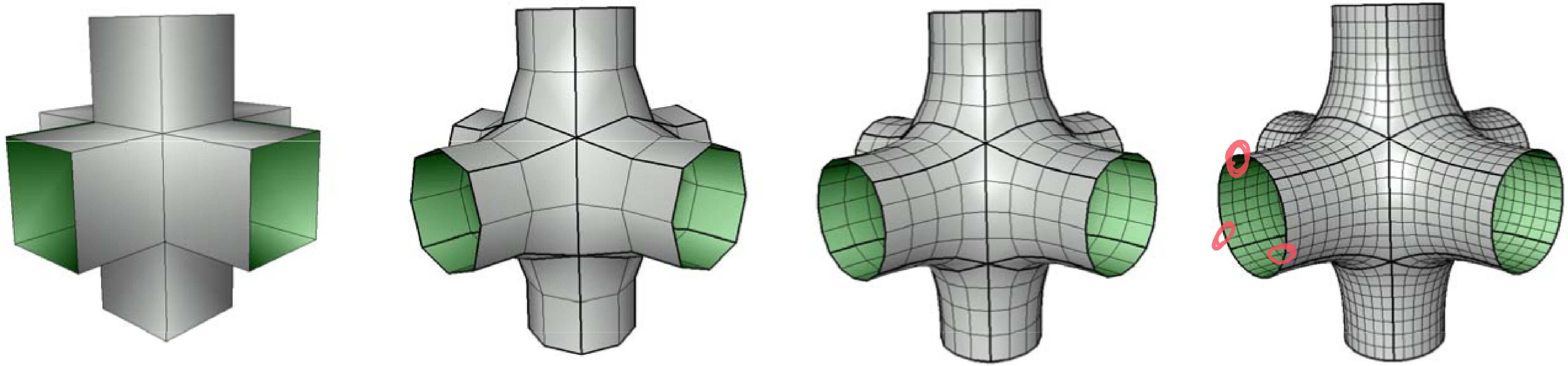


http://www.holmes3d.net/graphics/subdivision/



http://graphics.stanford.edu/courses/cs468-10-

Quads Example



http://graphics.stanford.edu/courses/cs468-10-fall/LectureSlides/10_Subdivision.pdf

What About Edges?

Edges depend only on edges:

- causes them to be "regular curves"

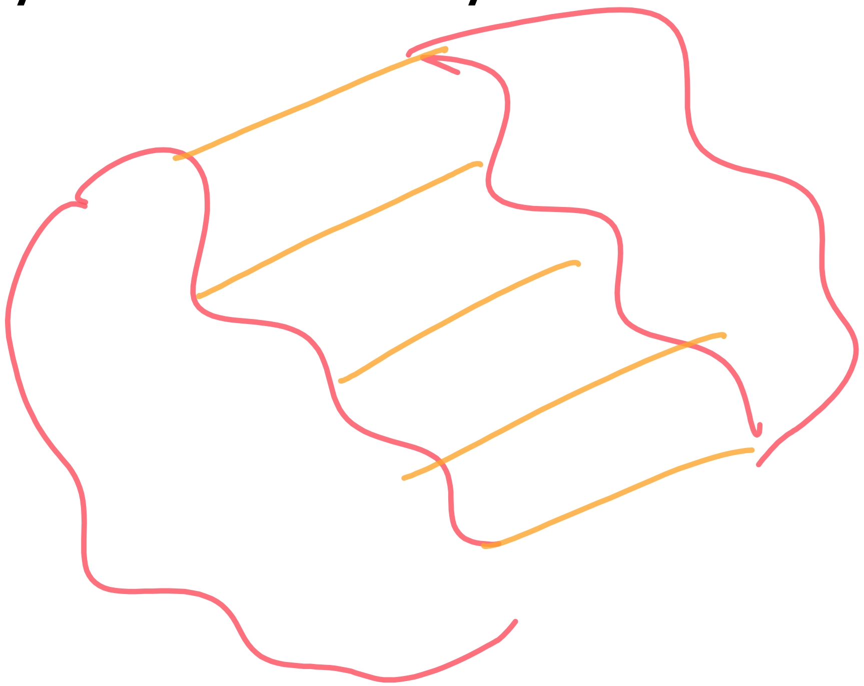
Good Tricks (1) ...

Creases - don't move points for some iterations

Good Tricks (2) … Cutting and Sewing

Put a curve inside of a surface (hole or edge)

Curves stay curves - on any surface!



Why do we like Catmull-Clark so Much?

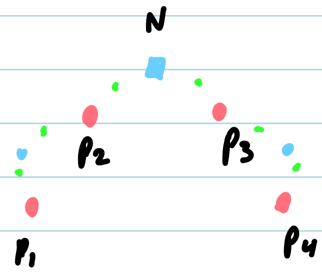
- Generalizes Cubic B-Splines
- Allows for stopping at any time
- Can compute exact normals (since B-Splines)
- Much easier than Non-Subdivision
- Not that hard to implement
  - requires mesh data structures for splitting and neighbor finding
- Made Popular by Pixar

(Smooth) Surfaces Review

- Surface vs. Solid Vs. Curve

- Not Free-Form
  - primitive shapes
  - generalized primitives (sweeps, lofts, ...)

- Free Form
  - Implicit
  - Parametric (and why not)
  - Subdivision (why and how)



$$N = \alpha P_1 + \beta P_2 + \beta P_3 + \alpha P_4$$

α ↑ $-w$

β ↑ $\frac{1}{2} + w$